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No. 831.

STEEL CONCRETE CONSTRUCTION.

BY GEORGE HILL, Assoc. M. Am. Soc. C. E.

PRESENTED APRIL 6TH, 1898.

WITH DISCUSSION.

It seems hardly necessary to write any prefatory note in connection with a paper on this subject, by reason of the number of papers already submitted, and the decided interest which is universal among engineers in the production of an economical fire-proof material for use under transverse stress. At short intervals there have been submitted, for the consideration of the engineer, combinations of plastic material and steel or concrete and steel, with the steel in the form of isolated rods and acting entirely in tension; combinations of concrete and steel, in which the steel acts entirely in tension and is distributed throughout the bottom area of the section, and combinations of concrete and steel in which the steel acts under transverse stress; each of which methods is claimed to possess certain advantages.

The present paper deals with that combination of concrete and steel in which the steel acts entirely in tension, and in which it is distributed through the bottom section of the slab. Theoretically, the best results are to be obtained from any combination of this charac-

ter where the entire compression is taken by the concrete, and where the metal used is so placed as to occupy the position of the extreme fiber on the tension side. Theoretically, it is also necessary, in order to develop the full strength of the metal, that it should be held in place by some means possessing greater strength than simple cohesion between the concrete and the steel. The difficulties to be encountered, in making any series of tests for the purpose of determining the laws governing the resistance of such combinations, were known by the fact that no formula had ever been suggested which even approximately accounted for the extremely high resistance of sections put in place in buildings and tested under concentrated loads. The companies interested in the manufacture of the materials used determined on having a series of tests made which should be devoted simply to the ascertaining of facts whether favorable or unfavorable, and the author was intrusted with the execution of these tests. The results obtained seem to be of sufficient value to justify their presentation to the Society, in order that they may be fully discussed and the real value of the combination may be generally known to engineers.

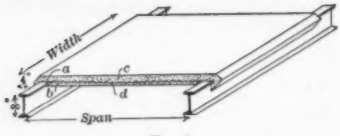

FIG. 1.

Slabs.—Fifty-six slabs of concrete and expanded metal of varying spans and thicknesses, both of concrete and of metal, but of practically uniform width, were erected on I-beam supports, of types shown in Figs. 1, 2, 3 and 4, and designated by the letters *A*, *B*, *C* and *D* in Table No. 3. These slabs were entirely independent of all others, so that the behavior of the material in each could be independently observed.

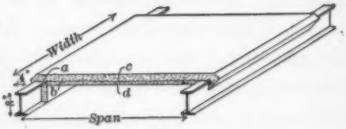

FIG. 2.

Cement.—The cements used were an American Portland and a slag cement. For the first of these the manufacturers claim a fineness of 90% on a No. 100 sieve, and 70% on a No. 200 sieve, and which actually showed a fineness of 96% on a No. 100 sieve, and 82% on a No. 180 sieve. A minimum tensile strength of neat cement, for seven days, of 400 lbs., and for 3 parts of sand to 1 part of cement, seven days, 150

lbs. was guaranteed. The tensile strengths under actual test exceeded the guaranteed amount considerably, the average of 30 tests giving 624 lbs. The slag cement is claimed to be ground to a fineness of 97% on a No. 180 sieve, and showed a fineness of 99.5% on a No. 100 sieve, and 98% on a No. 180 sieve. For neat cement, seven days, a tensile strength of 400 lbs. was claimed; for 3 parts of sand to 1 part of cement, seven days, 147 lbs. The slag cement ran very slightly below the amount claimed for neat cement. The fineness of the grinding, however, is a matter of very great importance, as will be shown later on.

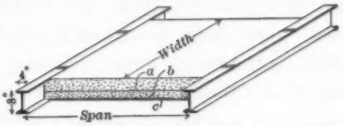

FIG. 3.

Sand.—Bank sand, clean and sharp, unscreened, and of varying sizes from moderately fine sand up to some pebbles the size of a white bean, was used.

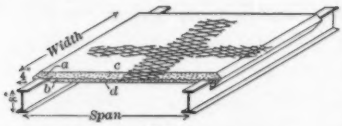

FIG. 4.

Cinders.—Ordinary steam cinders, varying in size from dust to pieces which would go through a ¾-in. ring, were used. Larger pieces were smashed by the shovel. The cinders were delivered by the barge-load for use in an adjoining building.

Stone.—The stone used was trap, broken to pass through a 1½-in. ring, and practically uniform in size. It was too large to give the best results for stone concrete, but was the only stone available.

Gravel.—The gravel used was clean washed, running in size from ½ in. to 1½ ins.

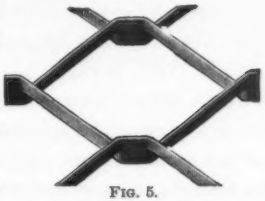
FIG. 5.

Metal.—The metal used was known as expanded metal (Fig. 5) made from high grade, low-carbon Bessemer steel, containing 0.008 carbon; unannealed specimens, tested in tension parallel with the grain, showing a tensile strength of about 65 000 lbs., an elongation of about 15% and an elastic limit of 30 000 lbs. Tests made on strands of expanded metal, after having been subjected to strain in the floor sections, showed an average ultimate strength of 54 240 lbs. The expanding is performed

by partially shearing through the sheet and extending the metal about 11 per cent. The strands which broke in the work in all cases broke at a point where the metal was bent to secure the diamond shape of the mesh. It is commercially designated by giving the gauge of the steel and the amount of displacement between the junction of the meshes, thus : No. 10, 3-in. mesh, designates an expanded metal made from No. 10 steel, in which the displacement of the bridge amounts to 3 ins., the axes of the diamond being $2\frac{3}{4}$ ins. and 6 ins. When expanded, this size and mesh weighs 0.56 lb. per square foot and has a sectional area per foot of width of 0.168 sq. in. and per inch of width of 0.014 sq. in. If the elastic limit then be taken at 30 000 lbs., the safe working strength per inch of width would be 200 lbs.

Expanded metal No. 4, 6-in. mesh, is made of No. 4 steel, in which the meshes are 5 ins. x 12 ins., its weight is 0.94 lb. per square foot, which equals 0.282 sq. in. of cross-section per foot of width, corresponding to 0.023 sq. in. per inch of width, or a working strength of 330 lbs. per inch of width.

In building the slabs, the expanded metal was usually put down in single sheets, with the long axis of the mesh at right angles to the beams, and so set as to be well covered by the concrete in the bottom of the slab. The sheets were sometimes lapped and sometimes doubled, and, when this was done, the only tying was that afforded by the concrete.

Testing Machine.—The testing machine was that described in a paper previously submitted by the author,* the only addition being that there were two gauges, both of the same range, one new and just calibrated, and the other the old one, but they checked together exactly. The plunger was in excellent condition, and practically without friction except at high loads, where the friction loss would be, in any event, inappreciable. The machine was handled by means of a detachable track in two parts, furnished by the McCabe Manufacturing Company, one section of the track always being in advance of the testing machine. The holding down was accomplished by means of two cast-steel wire rope straps passing around cross beams which ran underneath the supporting beams. The general arrangement is shown in Plate XVIII.

* *Transactions*, Vol. xxxiv, p. 544.

PLATE XVIII.
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Loading.—Except in a few isolated cases, the loads were applied by means of a 3-in. x 12-in. piece of yellow pine 3 ft. long, placed in the center of the slab, with blocking between it and the plunger. This load amounted to about 100 lbs., and was not allowed for in computing the loads resisted by the slabs in any case. As the loading was increased, deflections were measured by means of a rule held by an assistant stationed beneath the slab. The conditions of loading were practically that of a load concentrated over the entire width of the slab in the center, in such a manner as to avoid local injury to the concrete.

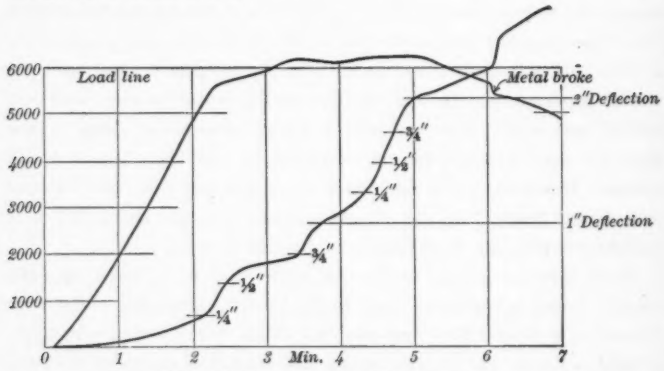
Mixing.—All the concrete was mixed by hand on a board platform. The proper amount of sand was first dumped on the platform, then the cement was placed on it and the two mixed dry by turning over once with shovels, then the stone, gravel or cinders was added and the whole mass turned over once while dry with shovels, and turned into a heap. It was then raked down from the heap into a bed, and the water sprinkled on. It was turned over by shovels as the water was added, and again shoveled into a heap. From the heap it was shoveled into a wheel-barrow, wheeled to the slab, dumped and spread. It was then well rammed with an ordinary 8-lb. rammer, and was usually brought up to a smooth surface. All the mixing was done by laborers who had been employed on similar work.

Each mixture usually made two slabs and a cylinder approximately 8 ins. in diameter and 15 ins. long. After the slabs were formed, and before there was any frost, they were covered with 2 ins. of sand. Before the testing began, the sand was swept off and the centers removed. The cinder concrete weighed 100 lbs. per cubic foot, and the stone and gravel concrete 140 lbs. per cubic foot, as an average.

Weather.—Mixing began on November 13th, and testing began on December 16th. During the interval the minimum temperature was 21° Fahr., on November 24th, and the maximum was 66 degrees. The days when the temperature went below the freezing point were, on November 18th, 30°; 23d, 23°; 24th, 21°; 25th, 32°; 28th, 26°, and on the 30th, 25°, on which day it froze all day long; December 1st, 27°; 2d, 28°; 3d, 29°, and 4th, 29 degrees. From these records it will be seen that the slabs were unhurt by frost, there being sufficient time before freezing weather for the cement to set.

General.—The conditions were such as to produce a slab of average manufacture, in which the action of both of the materials under test could be minutely observed.

Cylinders.—The cylinders before mentioned were made for the purpose of determining the actual compressive strength of particular mixtures of concrete employed, so that, used in conjunction with the known tensile strength of the metal and the known breaking strength of the combination, it would be practicable to apply various formulas for the purpose of comparison. A number of them were broken in the testing machine heretofore described, and the remainder in the testing machine of the New York University Laboratory, the use of which was kindly tendered by Professor Collins P. Bliss. The results are given in Table No. 3.



TEST NO. 13; LOADS AND DEFLECTIONS.

FIG. 6.

Tests.—The results of the general tests of the slabs are summarized in Table No. 3 in all cases except for Tests Nos. 57 to 60. The curve of loading was similar in its character to that shown in Fig. 6, which is the curve for Test No. 13.

Test No. 57.—This was a test of three special light beams and concrete, being as shown in Fig. 7. The expanded metal was laid in widths of 18 ins., measured in a direction parallel with the beams, and lapped 4 ins. It was of No. 10 metal, and the number of square inches in a direction at right angles to the tensile strains would be 1.33. The major axis of the diamond, however, was at right angles

with the line of strain, and, in consequence, the effective resistance of this metal diminished in the ratio of 2.2 to 1.

The composition of the concrete was 1 part American Portland, 2 parts sand, and 5 parts cinders. The load was applied to 1 sq. ft. in the center of the span immediately over the flanges of the central beam. The ratio of span to depth was such that a weight of 150 lbs. produced a deflection of $\frac{1}{4}$ in., and 3 000 lbs. produced a deflection of $\frac{1}{2}$ in. At 5 500 lbs. the deflection was $\frac{15}{16}$ in., and the side beams began to separate from the concrete. At 6 500 lbs. the top flanges of the center beam buckled under the load, and a deflection of $1\frac{7}{16}$ ins. was observed. This was the maximum strength. After the maximum

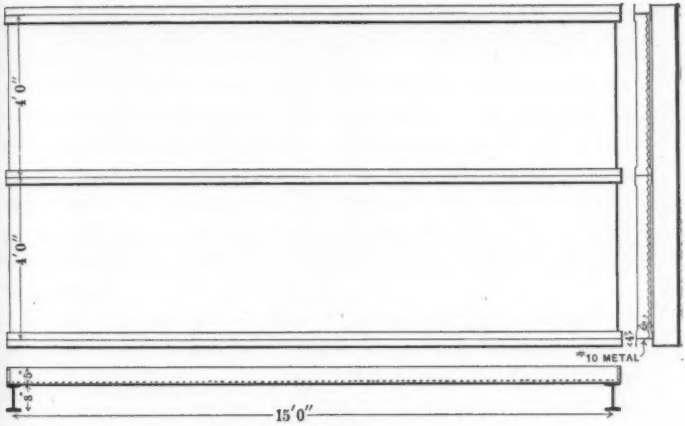


FIG. 7.

of 6 500 lbs. was reached, the increase in deflection and loss of resistance were very slow and uniform, reaching $3\frac{1}{8}$ ins., with a load of 5 800 lbs.

Test No. 58.—This was a test of a rectangular slab of concrete, 5 ins. thick, as shown in Fig. 8. The concrete was composed of American Portland, 1 part; sand, 2 parts, and cinders 5 parts. The metal was No. 10, in six pieces, and lapped 8 ins. longitudinally, and 20 ins. transversely; the total number of square inches of sectional area being 2.08. The load was applied to 1 sq. ft. in the center and was gradually increased. The elastic limit was at 10 500 lbs., and the deflection $\frac{11}{16}$ in. At 11 200 lbs., with a deflection of $\frac{13}{16}$ in., the corners of the slab began to lift,

and this lifting reached a maximum of $\frac{1}{4}$ in. at the corners, with a deflection of 2 ins. and a load of 11 000 lbs. While this pressure was maintained, a weight of 70 lbs. was twice dropped from a height of 2 ft. 4 ins. on the slab, immediately adjoining the loaded area, without

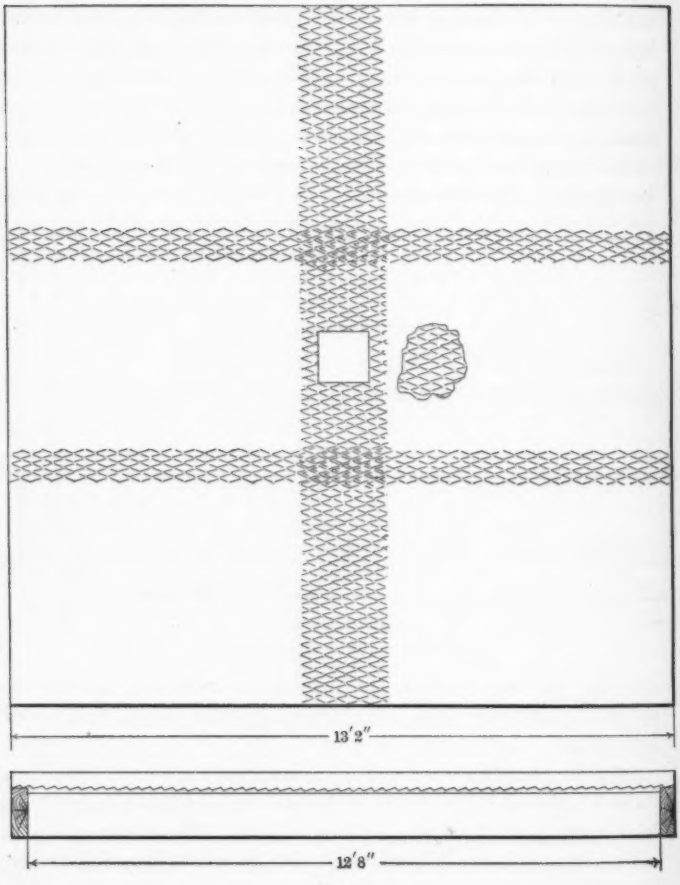


FIG. 8.

effect. A block $\frac{3}{4}$ sq. ft. in area was placed immediately adjoining the loaded area and a weight of 894 lbs. was dropped 1 ft. and decreased the pressure about 1 000 lbs., then it was twice dropped a distance of

15 ins., and twice a distance of 18 ins., the last time shearing an irregular hole of about 1½ sq. ft. in area, through the concrete, and decreasing the resistance to the plunger so that the load was in the vicinity of 4 500 lbs.

Test No. 59.—This was a test of a finished section of floor in the sugar house adjoining, and is shown in Fig. 9. The concrete was mixed in the proportion of American Portland cement, 1; sand, 2; cinders, 5. The top 2 ins. of the surface was mixed in the proportions of American Portland cement, 1; ¼-in. broken stone, 1, and sand, 1, and had a very smooth and handsome surface. The load was applied on an area of 1 sq. ft. The elastic limit was passed at a pressure of 35 000 lbs. The first crack appeared on the surface at a pressure of 37 600 lbs.; the maximum was reached with 37 750 lbs. The deflection was about 1½ ins., and then the failure became pronounced, cracks appearing in the top surface and roughly following the arc of a circle about 2 ft. distant from the point of application of the load for about one-half the area loaded, and, in addition, one crack running off diagonally at an angle of about 45° from the supporting beams. There were only slight cracks on the under surface.

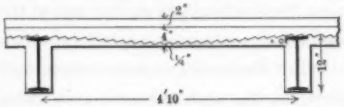


FIG. 9.

Test No. 60.—The section tested is shown in Fig. 10. Double expanded metal No. 10 was used. The concrete was the same as that used for Test No. 59. The load was applied on 1 sq. ft., and the slab resisted a total of 42 500 lbs. without sign of failure; the elastic limit was passed at about 35 000 lbs.

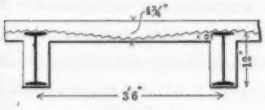


FIG. 10.

Observations.—The application of the load, in each case, was at a uniform rate. The deflections were practically at a uniform rate and were proportionate to the load up to the point where the elastic limit was reached. After the elastic limit was passed, they increased considerably; the bottom surface of the concrete began to show the compressing action of the strands of metal by breaking into small cracks and puckering up. Finally, the deflection became so great that small sections of the concrete between the strands fell off, and the strands of metal began to break. After the load curve showed a decided drop, the

pressure was relieved and usually the deflection decreased, the slabs showing some remaining elasticity. In no case was there any collapse of the section until ample warning had been given, and in no case was there a collapse of the section, except where the ratio of thickness of slab to span was far less than would ever be thought of in practice. In all cases the failure was identical in its character with that of an elastic slab supported at the ends and loaded in the center. The first crack, in almost every case, developed over the inner edge of the I-beam supporting the slab. The center of the slab usually crushed underneath the loading plank, but not at the edges of the plank. The bottom cracked across transversely in the center.

When the concrete was extremely hard, as in the stone and gravel mixtures, the slab did not curve between the point of application of the load and the point of support. In the cinder mixture there was sometimes a small but appreciable curvature. The supporting I-beams were steadied on top of the timbers with a couple of light spikes. In two or three cases there was a very slight rotation observed in the case of the soft concrete. In the remaining cases there was no movement of the beams. In some of the very long spans made with stone or gravel concrete the slabs rotated about the inner edge of the supporting beam flange and lifted at the outer edge of the flange an appreciable amount; in one case as much as $\frac{1}{4}$ in.

The general run of tests and the irregularity which exists among them, indicate the necessity of some more uniform method of mixing the concrete, if close calculations of the strength are to be made. The strengths shown by the mixtures in which the slag cement was employed are worthy of note, since the neat tensile strength tests showed a less strength than those in which American Portland was used, while in the actual use of the cement this difference disappears, and seems to indicate a considerable advantage due to fineness of grinding.

The tests were usually made in pairs, this being evident from a study of the tables, thus 1 and 2, 3 and 4, 5 and 6, etc., were made from the same mixtures of concrete and at the same time. It will be observed that the two slabs from the same mixture at times vary as much as two slabs made in the same proportions, but of different mixtures. It will be further observed that, in the case of the pair 5-6, in one case the metal was broken and the other not. The same thing is true of the pairs 7-8, 9-10, 17-18, 36-37, 40-41 and 44-45, thus indicat-

ing that the proportions of strength of concrete to strength of steel for this particular form of loading were practically correct, as in one case the steel and in the other case the concrete failed first. This is further emphasized by the tests 15–16, 31 and 33, in which there was an excess of metal and consequently no complete failure thereof; also by the tests 25–26, in which the compressive strength of the cinder concrete was exceptionally low and therefore the metal was not broken.

Constants. —Taking up the tests which seem to be fairly uniform, it is the author's judgment that a fair value for the ultimate compressive strength of a fairly well-mixed cinder concrete in the proportion of 1-2-5, or a thoroughly well-mixed cinder concrete in the proportion of 1-3-6, should be 400 lbs. per square inch. A stone or gravel concrete made of graded stones and thoroughly mixed by machinery should show a strength of at least 800 lbs. per square inch; in each case the strengths being for concretes 28 days old. The tensile strength of the concrete may be taken to be one-fifth of the compressive strength. In Table No. 1 is given a reduction of certain tests on cinder concrete slabs to a uniform condition of span, width and depth. Two points are noted for each test, first, that at which the deflection reached a point sufficiently great to cause a cracking in the plaster when applied to the under side; second, that at which the elastic limit was passed.

These tests show that a centrally applied load of 3 200 lbs. produces a deflection sufficient to crack plaster; a compressive strain of 300 lbs. per square inch in the concrete, and a tensile strain in the metal of 296 lbs. per inch of width, and a load of 4 580 lbs., reached the elastic limit of the combination, producing a compressive strain of 400 lbs. per square inch in the concrete, and a tensile strain of 601 lbs. per inch of width in the metal.

These results are obtained by the application of the formula hereinafter noted, on the assumption that the concrete and the steel will be strained proportionately within the elastic limit and up to the maximum at the elastic limit, and that if there is any reserve of strength it will exist only in the steel.

It is essential that in no case should the deflection produced by a load be sufficiently great to cause a cracking, either in the plaster or in the floor itself. It can also be seen from an examination of Table No. 3 that if the plaster does not crack, the concrete will not, since the deflection corresponding to the first crack is, in almost all cases, three

times as great as that required to crack the plaster. And, finally, it must be borne in mind that commercial conditions require the use of a cement so good that the contractor can remove his centers within a week or ten days, and in consequence any slab which will then bear its own weight is absolutely proof against collapse at any future time. Keeping these facts in view and considering that it is well to be conservative in all cases in the use of a new material, the author has used as his constants, for safe working strains of cinder concrete in compression 75 lbs. per square inch, and for stone concrete in compression 150 lbs. per square inch, which would correspond to a deflection, under uniformly distributed loads, of about one-quarter of that required to crack plaster, and to strains of about one-fifth of the elastic limit of the concrete.

Notation.—The following notation has been employed in the development of the formula:

C = Safe working strength of concrete per square inch, or 75 lbs. for cinders and 150 lbs. for stone.

T = Safe working strength of concrete in tension, per square inch $= \frac{C}{5}$.

S = Safe working strength of steel, per inch of width, or 200 lbs. for No. 10 metal, and 330 lbs. for No. 4 metal.

M = Bending moment of the external forces, in inch-pounds.

R = Resisting moment of the section, in inch-pounds.

l = Span, in inches.

t = Thickness of slab or section, in inches.

x = Distance from edge of slab or section to the neutral axis on the compression side.

y = Distance from the edge of slab or section to the neutral axis on the tension side.

W = Safe uniformly distributed load, per square foot.

w = Safe uniformly distributed load, per square inch.

Then when $W = 75$, $w = 0.52$;
$W = 125$, $w = 0.87$;
$W = 175$, $w = 1.22$;
$W = 250$, $w = 1.74$.

Formula.—The formula may be developed, within the elastic limit of the material, as follows:

Taking a section, it is evident that if the failure is as likely to occur on one side as on the other, as the tests indicate, the compres-

sion in the concrete above the neutral axis must equal the tension below that axis, due to the action of both the concrete and the steel. Taking any section of unit width, and adopting the notation given, the compression is represented by $\frac{C x^2}{3}$. The tension will be represented by $\frac{T y^2}{3} + S y$.

These two combined give $R = \frac{C x^2}{3} + \frac{T y^2}{3} + S y$ .....(Formula 1.)

It is evident that $R = \frac{2}{3} C x^2$ .....(Formula 2.)

These are practically correct when y is the distance from the neutral axis to the center of the expanded metal. They are within an extremely small percentage of being correct when the metal is imbedded in the

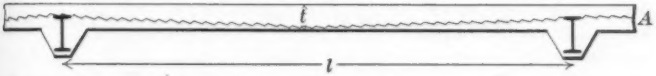


FIG. 11.

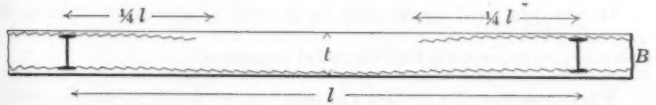


FIG. 12.

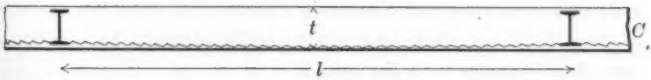


FIG. 13.

bottom of the concrete, and the result obtained by the formula will be practically correct and absolutely safe if the thickness of the slab be made $\frac{1}{4}$ in. in excess of the thickness determined by computation. Slabs may be built of any of the styles shown in Figs. 11, 12 and 13. In Fig. 11 the floor slab is in the condition of a continuous girder, supported at intervals. The moment of resistance of the section at the center is given by Formula 1. Over the points of support, since there is no steel, the moment may be expressed, with practical accuracy, by

$$R = \frac{C x^2}{3} + \frac{T y^2}{3} = 0.28\ T t^2 \text{ .....(Formula 3.)}$$

In Fig. 12 a condition exists similar to that of Fig. 11, except that, owing to the position of the concrete, there is no tensile strength to be

developed from it in the section over the point of support, and consequently Formula 1 is modified thus:

$$R = \frac{C x^2}{3} + S y \ldots\ldots\ldots\ldots\ldots\ldots \text{(Formula 4.)}$$

For conditions of uniformly distributed load and unit width, the moment of the external forces and the resistances of the sections in the center for the various cases is represented by:

$$M = \frac{w l^2}{8} = \frac{C x^2}{3} + \frac{T y^2}{3} + S y = R \ldots\ldots \text{(Formula 5.)}$$

$$M = \frac{w l^2}{24} = \frac{C x^2}{3} + \frac{T y^2}{3} + S y = R \ldots\ldots \text{(Formula 6.)}$$

and the sections over the points of support are represented by

$$M = \frac{w l^2}{12} = 0.28\ T l^2 = R \ldots\ldots\ldots\ldots\ldots \text{(Formula 7.)}$$

$$M = \frac{w l^2}{12} = \frac{C x^2}{3} + S y = \tfrac{2}{3}\ C x^2 = R \ldots \text{(Formula 8.)}$$

In the practical application it is well to remember that in all cases $\frac{C x^2}{3}$ represents half the total resistance.

When concrete of a certain character is used, it is a simple matter to assign safe working values for C, T and S, and then assign values for x and solve for y, thus obtaining a curve from which the thickness of the slab for any given load can be easily obtained by equating M with R, as obtained from Formula 2.

In this way the tables of requirements for safe uniformly distributed loads have been computed. It is to be particularly observed that no formulas are given for the resistance to concentrated loads. This is for the reason that concentrated loads so rarely occur, and the resistance to them is so great as to render it practically a waste of time. This resistance may be calculated on the assumption that the resistance will equal that of a circular disc, of a diameter equal to the span, fixed on all edges and loaded in the center.

TABLE No. 1.

Test No.	Load producing deflection sufficient to crack plaster.	Elastic limit.		Test No.	Load producing deflection sufficient to crack plaster.	Elastic limit.	
3	3.64	4.13	.....	37	2.90	.....	5.60
4	2.95	5.40	.....	48	3.10	.....	5.40
13	3.82	5.20	.....	49	3.40	.....	7.00
14	3.30	4.40	.....	50	3.40	5.00	.....
27	3.10	4.00	.....	51	3.40	4.40	.....
28	2.85	3.80	.....	52	3.00	.....	3.60
34	2.80	4.30	.....	53	3.50	4.80	.....
35	2.90	.....	5.60	54	3.10	4.70	.....
36	3.10	3.90	.....				
				Average.	3.20	4.58	5.44

Cinder concrete only. Loads in thousands of pounds.
Last column shows elastic limit, where conditions affected this limit, but not the plaster limit.

TABLE No. 2.—Characteristics of Floor Slabs for Carrying the Designated Live Loads. Safe Working Strength of Cinder Concrete 75 Lbs. in Compression, 15 Lbs. in Tension. Of Stone Concrete, 150 Lbs. and 30 Lbs., Respectively. Of No. 10 Metal, 200 Lbs. per Inch-Width. Of No. 4 Metal, 330 Lbs. per Inch-Width.

75 lbs. per square foot, live load, uniformly distributed.

Span	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Concrete	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*S*	*S*	*S*	*S*	*S*	*S*
Top	.....	.....	.....	.....	.....	4	4	4*D*	4*D*	4*D*	4*D*	4*D*	4*D*	4*D*	4*D*
Bottom	10	10	10	4	4	10	10	10	10	10	10	10	10	10	10
Thickness	2.75	3.5	5	4.75	5.5	5	6	5.75	6.5	5.5	6.0	6.75	7.5	8.5	9.25
Type	*A C*	*A C*	*A C*	*A C*	*A C*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*

125 lbs. per square foot, live load, uniformly distributed.

Span	3	4	5	6	7	8	9	10	11	12	13	14	15
Concrete	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*S*	*S*	*S*
Top	.....	.....	.....	.....	.....	4	4	4	4*D*	4*D*	4*D*	4*D*	4*D*
Bottom	10	10	10	4	4	10	10	10	10	10	10	10	10
Thickness	2.25	3	4.25	4	5	5	5.75	7	6.75	7.5	6.5	7.5	8
Type	*A C*	*A C*	*A C*	*A C*	*A C*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*

175 lbs. per square foot, live load, uniformly distributed.

Span	3	4	5	6	7	8	9	10	11	12	13	14	15
Concrete	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*C*	*S*	*S*	*S*	*S*	*S*
Top	.....	.....	.....	.....	.....	4	4	4*D*	4*D*	4*D*	4*D*	4*D*	4*D*
Bottom	10	10	10	4	4	10	10	10	10	10	10	10	10
Thickness	2.5	3.5	4.0	5	6.25	6	7	8	6	7	8	9	10
Type	*A C*	*A C*	*A C*	*A C*	*A C*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*

250 lbs. per square foot, live load, uniformly distributed.

Span	3	4	5	6	7	8	9	10	11	12
Concrete	*C*	*C*	*C*	*S*	*S*	*C*	*C*	*C*	*S*	*S*
Top	.....	.....	.....	.....	.....	4	4*D*	4*D*	4*D*	4*D*
Bottom	10	10	10	4	4	10	10	10	10	10
Thickness	2.75	3.75	4.75	5.25	8	7	7.25	8.25	7.75	8.75
Type	*A C*	*A C*	*A C*	*A C*	*A C*	*B*	*B*	*B*	*B*	*B*

TABLE No. 3.

Test number.	Age, in days.	Span.	Width.	a.	b.	c.	d.	Square inches of metal in cross-section.	Load causing deflection that would crack plaster.	Deflection.	Elastic limit.	Deflection.	Load producing first crack.	Deflection.	Load at which metal broke.	Deflection.	Maximum load.	Deflection.
1	18	60	42.0	....	....	3.25	....	0.58	.....	....	....	....	....	....	....	....	2.8	4.0
2	31	60	42.5	1.88	1.50	3.50	0.38	0.58	1.50	0.16	2.5	0.62	2.4	0.54	not	....	3.0	0.5
3	31	60	42.75	1.65	1.31	2.88	0.75	0.58	3.0	0.16	3.4	0.31	3.25	0.25	3.50	1.50	4.1	0.69
4	31	60	42.0	2.00	1.69	3.25	0.75	0.58	2.75	0.16	5.0	0.81	4.00	0.37	5.00	1.00	5.0	1.00
5	30	60	42.0	1.94	1.50	3.45	0.05	0.58	2.10	0.16	2.5	0.33	2.90	0.50	not	....	3.1	1.00
6	31	60	42.0	1.87	1.31	3.30	0.20	0.58	2.5	0.16	3.0	0.37	2.9	0.31	3.0	1.50	3.4	1.06
7	30	60	41.5	2.0	1.4	3.50	0.25	0.58	2.6	0.16	3.4	0.31	3.5	0.37	3.6	3.50	3.6	3.00
8	30	60	42.5	1.85	1.69	3.50	0.25	0.58	2.3	0.16	3.0	0.56	3.0	0.69	not	....	3.0	1.5
9	30	60	41.0	1.88	1.31	3.50	0.12	0.58	3.5	0.16	4.0	0.28	4.2	0.50	3.5	2.0	4.2	0.44
10	30	60	42.0	2.06	1.44	3.25	0.50	0.58	3.8	0.16	4.6	0.50	4.6	0.50	not	....	4.6	0.50
11	25	60	42.0	1.88	1.31	3.75	0.06	0.58	5.0	0.16	5.2	0.22	6.05	0.37	4.0	0.94	6.05	0.37
12	25	60	42.0	2.00	1.44	4.44	0.06	0.58	3.5	0.16	4.0	0.37	4.2	0.50	4.25	2.75	4.4	2.0
13	28	60	42.5	2.00	1.31	3.75	0.25	0.97	4.1	0.16	5.6	0.75	5.5	2.25	5.5	2.25	6.15	0.88
14	28	60	43.0	1.94	1.75	4.38	0.12	0.97	4.1	0.16	5.5	0.50	5.55	0.88	5.9	1.31	6.0	1.81
15	28	60	42.0	1.81	1.75	4.75	0.0	1.16	5.1	0.16	9.3	0.99	9.3	0.69	not	....	9.3	0.69
16	28	60	42.0	1.75	1.50	3.94	0.0	1.16	5.6	0.16	9.3	0.62	....	....	not	....	9.8	1.50
17	30	60	42.0	1.81	1.62	3.80	0.07	0.58	3.9	0.16	4.4	0.25	4.5	0.38	not	....	4.8	0.75
18	30	60	42.0	1.94	1.50	3.50	0.50	0.58	4.9	0.16	6.5	0.38	6.5	0.38	5.8	1.31	6.5	0.50
19	30	60	43.0	4.25	3.68	0.06	....	0.58	2.4	0.16	5.0	0.56	....	....	5.0	0.56	5.0	0.56
20	30	60	42.0	4.75	3.0	0.25	....	0.58	2.9	0.16	5.0	0.75	5.1	1.25	5.0	2.38	5.10	1.25
21	30	60	41.0	4.0	3.87	0.12	....	0.58	3.1	0.16	4.5	0.37	4.7	0.56	4.30	2.25	4.85	1.0
22	30	60	42.0	4.0	3.87	0.12	....	0.58	3.4	0.16	5.48	0.50	5.48	0.50	5.0	1.75	5.6	0.75
23	33	60	40.0	4.0	3.75	0.25	....	0.58	4.4	0.16	7.9	0.44	....	....	8.6	1.12	8.6	0.88
24	33	60	40.5	4.25	3.25	0.50	....	0.58	3.4	0.16	6.0	0.56	....	....	8.0	2.00	8.30	2.50
25	33	60	41.0	4.37	3.25	0.37	....	0.58	2.9	0.16	4.5	0.75	....	....	not	....	5.60	2.62
26	33	60	42.5	4.00	3.88	0.12	....	0.58	3.1	0.16	6.0	0.75	....	....	not	....	7.2	2.50
27	32	72	42.0	2.12	1.12	4.12	0.12	0.60	3.0	0.20	3.9	0.50	....	....	3.8	0.31	3.95	0.94
28	32	72	42.0	2.00	1.19	3.68	0.25	0.60	2.5	0.20	3.9	0.62	4.0	0.88	3.2	1.81	4.0	0.88

TABLE No. 3—(*Continued*).

Test number.	Final load.	Deflection.	Permanent deflection.	Type of slab.	Cylinders. Diameter.	Square inches, sect. area.	Total load.	Load per square inch.	Length.	Remarks.—All loads in thousands of pounds. All dimensions and deflections in inches. Tests 1 to 33, both inclusive, made with American Portland cement. Tests 34 to 56, both inclusive, made with slag cement.
1	....	....	....	*A*	8.48	56.7	16.5	292	.....	Cinders, 1-3-6.
2	2.9	2.00	....	*A*	....	....	.....	....	.....	Cinders, 1-3-6. Good top surface, mixed very moist.
3	3.2	2	....	*A*	....	....	.....	....	.....	Cinders, 1-2-5. Mixed very moist.
4	3.0	3	....	*A*	....	....	.....	....	.....	Cinders, 1-2-5. Mixed very moist. Small section of bottom near center and below metal fell out.
5	2.0	2	....	*A*	8.53	57.2	22.7	398	16.0	Stone, 1-3-6. Mixed a little too dry. N. edge sheared down. Top crushed in.
6	2.7	2	....	*A*	....	....	.....	....	.....	Stone, 1-3-6. Mixed about right. Top crushed. Bottom opened in one crack.
7	3.1	4	....	*A*	8.38	55.2	34.5	625	.....	Stone and gravel graded, 1-2-10. Concrete moist. Top surface rough. Top crushed, bottom spaced and cracked.
8	2.5	3	....	*A*	....	....	.....	....	.....	Stone and gravel graded, 1-2-10. Concrete moist. Top surface rough. Top crushed, bottom spaced and cracked.
9	3.5	2	....	*A*	8.48	56.7	52.0	917	.....	Gravel, 1-3-6. Concrete a little wet. Fine top surface. Crushed, bottom cracked.
10	4.0	2	....	*A*	....	....	.....	....	.....	Gravel, 1-3-6. Concrete a little wet. Fine top surface. Crushed, bottom cracked.
11	4.0	0.94	....	*A*	8.17	52.3	46.0	880	12.25	Gravel, 1-2-7. Fine top surface. Crushed 15 ins. N. of center on diagonal line.
12	3.75	2.81	2.0	*A*	....	....	.....	....	.....	Gravel, 1-2-7. Good surface. Section of bottom below metal dropped out.
13	5.0	2.75	2.25	*B*	....	....	.....	....	.....	Cinders, 1-2-5. Good surface. Top crushed. Edges lifted, bottom cracked.
14	5.8	2.31	1.62	*B*	....	....	.....	....	.....	Cinders, 1-2-5. Good surface. Top crushed. Edges lifted, bottom cracked.
15	9.0	1.06	0.75	*B*	....	....	.....	....	.....	Gravel, 1-3-6. Fair surface. Top cracked diagonally.
16	9.5	2.31	1.31	*B*	....	....	.....	....	.....	Gravel, 1-3-6. Fair surface. Top crushed near center.
17	3.8	2.56	2.06	*B*	....	....	.....	....	.....	Stone, 1-2.4-4.1. Fair surface. Top crushed. Cement fell off bottom below metal.
18	5.0	2.38	2.0	*B*	....	....	.....	....	.....	Stone, 1-2.4-4.1. Fair surface. Top crushed. Cement fell off bottom below metal.
19	3.5	1.12	1.12	*C*	8.47	56.3	28.35	503	.....	Stone, 1-2.4-4.1. Filling of Rosendale, 1; sand, 2; cinders, 10; about twelve days old.
20	4.5	4.5	4.12	*C*	....	....	.....	....	.....	Stone, 1-2.4-4.1. Filling of Rosendale, 1; sand, 2; cinders, 10; about twelve days old.
21	4.0	3.25	2.88	*C*	....	....	.....	....	.....	Gravel, 1-3-6. Filling like Test 19. Loaded on 1 sq. ft. Concrete fell from bottom.
22	4.0	2.0	1.75	*C*	....	....	.....	....	.....	Gravel, 1-3-6. Filling like Test 19.
23	6.6	1.44	1.12	*C*	8.31	54.2	16.5	305	.....	Cinders, 1-2-5. Filling like Test 19. Filling somewhat frozen.
24	7.5	3.00	2.56	*C*	....	....	.....	....	.....	Cinders, 1-2-5. Filling like Test 19. Filling somewhat frozen.
25	5.0	4.25	....	*C*	8.38	55.2	13.53	246	15.5	Cinders, 1-3-6. Filling like Test 19. Bottom much scaled. Top not frozen.
26	6.0	3.62	3.25	*C*	....	....	.....	....	.....	Cinders, 1-3-6. Filling like Test 19. Bottom much scaled. Top not frozen.
27	2.50	1.25	0.69	*A*	8.31	54.2	26.5	490	.....	Cinders, 1-2-5. Metal in two pieces, with longitudinal lap of 1½ ins.
28	2.5	2.18	1.81	*A*	....	....	.....	....	.....	Cinders, 1-2-5. Metal in two pieces, with longitudinal lap of 2 ins. Good surface, bottom cracked.

TABLE No. 3—(*Continued*).

Test number.	Age, in days.	Span.	Width.	a.	b.	c.	d.	Square inches of metal in cross-section.	Load causing deflection that would crack plaster.	Deflection.	Elastic limit.	Deflection.	Load producing first crack.	Deflection.	Load at which metal broke.	Deflection.	Maximum load.	Deflection.
29	33	96	42.0	2.12	1.19	4.25	0.13	0.65	1.8	0.27	2.3	0.50	....	....	2.5	1.06	2.5	1.06
30	33	96	41.5	2.06	1.13	4.26	0.25	0.66	1.8	0.27	2.4	0.62	....	....	2.4	0.62	2.4	0.62
31	32	120	42.0	2.06	1.31	4.26	0.25	1.38	2.1	0.31	3.5	1.00	....	....	not	....	3.7	1.50
32	32	108	41.0	2.06	1.19	4.44	0.12	0.65	2.4	0.30	3.0	0.56	....	....	3.0	2.0	3.5	1.00
33	33	120	42.0	2.19	1.38	4.13	0.37	1.94	2.70	0.39	4.0	1.00	4.25	1.38	not	....	4.3	1.50
34	24	60	42.5	2.56	1.12	3.70	0.12	0.58	3.0	0.16	4.5	0.75	3.4	0.25	4.65	1.69	4.65	1.69
35	24	60	42.0	2.69	1.19	3.25	0.50	0.58	2.7	0.16	5.25	0.81	4.5	0.44	4.7	3.25	5.25	0.81
36	30	60	42.0	1.81	1.25	3.37	0.12	0.58	3.0	0.16	3.7	0.50	3.0	0.19	3.8	3.25	4.25	2.37
37	30	60	42.5	1.81	1.25	3.12	0.37	0.58	2.6	0.16	5.0	0.75	3.5	0.37	not	...	5.05	1.0
38	30	60	42.5	1.88	1.56	3.7	0.05	0.58	3.6	0.16	5.95	0.75	5.0	0.33	5.5	1.12	5.95	0.75
39	30	60	43.0	1.81	1.12	3.25	0.25	0.58	2.6	0.16	4.5	0.75	4.0	0.44	4.3	1.69	4.5	1.50
40	30	60	42.0	1.94	1.31	2.94	0.87	0.58	3.1	0.16	5.2	0.50	4.5	0.31	4.75	1.25	5.2	0.75
41	30	60	42.0	1.81	1.50	3.31	0.75	0.58	3.6	0.16	6.0	0.50	6.0	0.62	not	....	6.0	0.50
42	31	60	42.0	1.88	1.25	3.81	0.25	0.58	3.1	0.16	5.5	0.50	5.6	0.56	5.0	0.38	5.6	0.56
43	31	60	42.0	1.88	1.31	2.88	0.50	0.58	3.2	0.16	4.5	0.44	4.50	0.50	4.0	1.25	4.6	1.00
44	31	60	42.0	1.69	1.56	2.94	0.87	0.58	3.1	0.16	4.75	0.75	3.9	0.33	not	....	4.90	0.81
45	31	60	43.0	2.0	1.56	3.69	0.25	0.58	4.6	0.16	5.40	0.25	5.4	0.25	5.5	2.5	5.7	2.0
46	30	60	45.5	4.5	3.25	0.25	....	0.58	3.6	0.16	5.6	0.75	5.5	0.44	4.6	3.5	5.75	1.00
47	30	60	43.0	4.37	3.37	0.25	....	0.58	4.6	0.16	5.8	0.50	5.5	0.28	6.1	0.87	6.1	0.87
48	30	60	41.0	4.50	3.25	0.25	....	0.58	2.6	0.16	5.0	0.75	....	....	5.4	1.87	5.4	1.62
49	30	60	43.5	4.50	3.0	0.50	....	0.58	2.9	0.16	6.0	0.61	....	....	5.5	2.75	6.2	1.50
50	31	72	42.5	2.00	1.38	4.12	0.25	0.58	3.3	0.20	4.9	0.50	....	....	4.5	2.25	5.0	0.62
51	31	72	42.5	2.00	1.25	4.32	0.12	0.58	3.5	0.20	4.5	0.44	....	....	4.5	1.50	4.60	1.0
54	32	96	43.0	2.00	1.12	3.88	0.25	0.59	2.1	0.27	2.5	0.30	....	....	2.5	1.25	3.0	0.62
53	28	96	42.5	2.12	1.12	3.38	0.37	0.65	2.1	0.27	2.9	0.50	2.9	0.5	2.5	1.00	2.9	0.50
54	28	120	42.0	2.12	1.25	3.0	0.25	0.61	1.3	0.30	2.0	1.00	1.8	0.75	1.5	2.75	2.15	1.25
55	28	120	42.0	2.00	1.06	4.06	0.13	0.64	1.2	0.30	1.9	1.00	1.9	2.0	1.6	2.25	2.0	1.5
56	33	120	42.0	2.19	1.37	3.87	0.06	1.16	2.5	0.30	4.2	0.88	4.5	1.0	4.5	2.0	4.75	1.50

TABLE No. 3—(*Continued*).

Test number.	Final load.	Deflection.	Permanent deflection.	Type of slab.	Cylinders. Diameter.	Square inches, sect. area.	Total load.	Load per square inch.	Length.	Remarks.—All loads in thousands of pounds. All dimensions and deflections in inches. Tests 1 to 33, both inclusive, made with American Portland cement. Tests 34 to 56, both inclusive, made with slag cement.
29	....	....	....	*D*	8.54	57.2	26.5	464	.....	Cinders, 1-2-5. Metal in four pieces, lapping 7½ ins. longitudinally and 1 ft. transversely. Good surface.
30	....	....	....	*D*	....	....	.....	....	.....	Cinders, 1-2-5. Metal in four pieces, lapped 6 ins. longitudinally and transversely.
31	3.1	2.56	1.62	*D*	....	....	.....	....	.....	Cinders, 1-2-5. Metal in eight pieces, doubled, lapped 6 ins. longitudinally, 18 ins. transversely. Good surface.
32	2.5	2.75	2.38	*D*	8.38	55.2	32.5	590	.....	Cinders, 1-2.4-6. Metal in four pieces, lapped 6 ins. longitudinally, 18 ins. transversely. Load applied at two points, dividing span into three equal parts.
33	3.00	3.62	3.00	*D*	....	....	.....	....	.....	Stone, 1-3-5. Metal doubled in four pieces, lapped 18 ins. transversely. Good top surface, crushed.
34	4.0	2.5	....	*A*	....	....	.....	....	.....	This and all remaining tests made with slag cement. Cinders, 1-2.4-4.8. Good surface. Part of bottom below metal fell off.
35	4.5	3.5	....	*A*	....	....	.....	....	.....	Cinders, 1-2.4-4.8. Good surface. Load on 1 sq. ft. Top crushed, bottom cracked.
36	3.8	3.25	....	*A*	8.63	58.5	20.0	342	.....	Cinders, 1-1.7-4.2. Bottom blistered. Top surface good and crushed. Metal strands drawn together.
37	3.3	3.5	....	*A*	....	....	.....	....	.....	Cinders, 1-1.7-4.2.
38	3.0	1.31	0.88	*A*	8.35	54.8	40.0	730	.....	Stone, 1-2.4-4. Rough top surface.
39	3.8	3.0	1.56	*A*	....	....	.....	....	.....	Stone, 1-2.4-4. Top slightly crushed.
40	4.2	1.75	1.37	*A*	8.48	56.7	35.0	618	.....	Gravel, 1-3-6. Concrete a little wet; fine top surface. Crushed on top, cracked below.
41	5.00	2.71	2.31	*A*	....	....	.....	....	.....	Gravel, 1-3-6. Concrete a little wet; fine top surface. Crushed on top, cracked below.
42	3.5	1.06	0.75	*A*	....	....	.....	....	.....	Gravel, 1-2.4-4.9. Fair top surface. Top crushed.
43	3.5	2.25	1.94	*A*	....	....	.....	....	.....	Gravel, 1-2.4-4.9. Fair top surface. Top crushed.
44	4.0	2.0	1.62	*A*	8.35	54.8	40.0	730	.....	Gravel, 1-1.6-6.5. A little moist. Fair top surface.
45	5.0	2.66	2.31	*A*	....	....	.....	....	.....	Gravel, 1-1.6-6.5. A little moist. Fair top surface.
46	3.75	3.87	3.56	*C*	8.28	53.8	35.0	650	.....	Gravel, 1-2.4-4.8. Filling like Test 19. Top crushed in.
47	5.3	1.87	1.61	*C*	....	....	.....	....	.....	Gravel, 1-2.4-4.8. Filling like Test 19. Top crushed in.
48	4.8	2.75	....	*C*	8.48	56.7	18.72	330	15.5	Cinders, 1-1.6-4. Filling like Test 19.
49	5.0	3.19	2.75	*C*	....	....	.....	....	.....	Cinders, 1-1.6-4. Filling like Test 19.
50	....	....	1.75	*A*	8.47	56.3	43.0	765	15.25	Cinders, 1-1.6-4.
51	3.9	2.94	1.56	*A*	8.47	56.3	43.0	765	15.25	Cinders, 1-1.6-4. Bottom cracked.
52	....	....	....	*A*	....	....	.....	....	.....	Cinders, 1-1.6-4. Good surface.
53	....	....	....	*D*	....	....	.....	....	.....	Cinders, 1-1.6-4. Good surface. Metal in four pieces, lapped, 14 ins. transversely, and 4 ins. longitudinally.
54	....	....	....	*D*	....	....	.....	....	.....	Cinders, 1-1.6-4. Good surface. Metal in four pieces, lapped 15 ins. transversely. Edges lifted.
55	1.0	3.0	....	*D*	....	....	.....	....	.....	Cinders, 1-1.6-4. Good surface. Metal in four pieces, lapped, 15 ins. transversely. Top crushed. Edges lifted.
56	4.00	2.44	1.75	*D*	8.35	54.8	35.27	645	.....	Stone, 1-2.4-4. Metal doubled in four pieces, lapped 24 ins. transversely. Fair surface.

DISCUSSION.

Mr. Lesley. R. W. LESLEY, Assoc. Am. Soc. C. E.—The use of cinder in concrete seems to bear on this subject. The general cinder, from either anthracite or bituminous coal, contains a considerable amount of sulphur, and there are many cases where this substance has been known to rust iron pipe and cause all sorts of trouble. How far have these experiments demonstrated the action of the cinder upon this iron, which is depended upon for tension in these various spans? This is a vital question, if the concrete is loaded beyond its maximum and is depending upon the metal beneath it, and if the metal in turn is suffering from the action of the sulphur on the iron. It is a well-known fact that ashes have injured iron around which they have been placed. Around the works of the speaker's company, pipes which have been imbedded in ashes or ordinary cinders have been corroded. For that reason the speaker would not advise the use of cinders with iron. He also doubts the wisdom of using a limestone in concrete for fire-proofing. If the floors in a large building are made of limestone concrete and a fire occurs, might not the limestone be converted into lime, or something approaching lime, and this lime be slacked when the hose is turned on? This is a question which has never been determined, and the speaker merely mentions it as a possibility. It would not apply, of course, when slag was used or any rock formed in Nature by the action of fire.

The number of calculations of the strength of concrete beams seems to be rather meager. For instance, in one of the discussions of this paper it is stated that in the literature on this subject there are only two calculations of the strength of iron and steel and concrete beams. The literature of France is full of such calculations. One publication, *Le Ciment*, is devoted almost entirely to iron and concrete construction, which has grown very rapidly in France. Each issue contains calculations relative to every conceivable use of concrete construction. The same may be said of the proceedings of the German Society of Cement Manufacturers. They have, in their publications, a great number of calculations of the strength of these materials for different uses.

The idea of using expanded metal in the way described in the paper is interesting from the fact that the good results of keying plaster to ordinary lath are well known. By the use of expanded metal, not only is the strength of the cement utilized, but the keying of the cement to the back of the metallic lath is also obtained. Expanded metal is a very good material for this purpose, but it should be used with the best concrete obtainable and not with an inferior or doubtful material which might induce a possible failure.

Mr. Tomkins. CALVIN TOMKINS, Assoc. Am. Soc. C. E.—A large part of the concrete used in New York City is made with limestone. The speaker believes that if there is any question in the matter at all, the use of limestone, rather than proving a disadvantage, would prove advantageous, as against fire. Granite and trap-rock are likely to be adversely affected by fire, since the interior moisture in these stones expands and bursts them. However, in the case of limestone subjected to great heat, the carbonate of lime is converted into lime from without inwards, and in the course of this process a large amount of heat is used up in effecting a chemical change. Plaster-of-paris is used as a fire-proofing material for the same reason. When heated, the water of crystallization is carried off, and heat is absorbed in that way. In the case of limestone, the action is similar, and long before the limestone itself has been converted into lime, the iron in the building will have lost its properties under continued red heat. To the extent that iron construction is covered with limestone concrete, it is to that extent covered by a fire-proof covering. In the opinion of the speaker, a fire of sufficient intensity to convert carbonate of lime into lime would destroy the whole structure before it would materially injure the concrete alone.

Mr. Merritt. J. S. MERRITT, Esq.—In cases where pipes are imbedded in cinder concrete, if the mixture has been properly made, the cement covers up the sulphur so that it does not come in contact with the metal. The speaker had seen steel which had been imbedded in cinder concrete for about five years, and when taken out was as clean as when it was put in. The amount of sulphur in the cinder was not determined.

The effect of limestone on iron has been discussed recently in technical papers, and the conclusion has been that if the concrete is properly mixed the cement coats every particle of the sand, cinder or whatever material is used, and protects the iron from attack.

Mr. Washburn. F. S. WASHBURN, M. Am. Soc. C. E.—Comment has been made on the variable strengths of the cinder slabs used in the author's tests. The table of tests shows that a great many of the slabs failed by crushing on top. This crushing was confined probably to a small area in which the maximum strain would occur. Theoretically, it would occur along a line or at a point. That fact, together with the method of mixing, which the speaker thought was not as careful as is customary for first-class work, might explain this variation. It seems that the dry cement and sand were mixed by one turning with the shovel. Then the stone, gravel and cinders were added. The mixture of sand, cement and stone, gravel and cinders, was turned only once. It was then raked down from a heap into a bed. The speaker has noticed in some cases that raking down results not in a mixing, but in a partial separation of the materials of the concrete, particularly if the mixture is still dry, being due to the different sizes of the materials, the heavy particles going to the bottom.

Mr. Washburn. After the materials were wet, they were then turned only once. Is it not conceivable that in the material, or in the same batch or smaller batches made at different times, there were masses or spots where possibly there was very little cement? Those batches with such spots may have been placed in the slabs at the point of maximum stress, and in such cases the tests would show the slabs to be comparatively weak.

Mr. Dunham. H. F. DUNHAM, M. Am. Soc. C. E.—The elaborate formulas, equations and assumptions that are made in regard to this subject are not held in very high regard by the speaker. Many engineers have seen so-called good work, where concrete and steel were associated, taken down after it had been in place a number of years, and few of them would recommend introducing for permanent construction any form in which it would be necessary to depend upon No. 10 steel in shreds imbedded in concrete. Permanent loads, in excess of those which might be properly applied to concrete of the thickness described and without reference to the steel introduced in it, should not be placed on such construction.

Mr. Merriman. MANSFIELD MERRIMAN, M. Am. Soc. C. E.—An important question, which has not been touched upon in the paper, relates to the percentage of strength given to the beam by the introduction of expanded metal. Were any tests made of slabs otherwise similar in character, one kind containing the expanded metal and the other not? If so, what was the increase in strength due to the expanded metal?

Mr. O'Rourke. JOHN F. O'ROURKE, M. Am. Soc. C. E.—Some time ago plans for floors similar to those described in the paper were submitted to the speaker for his examination. The floors were intended for a building, the plans of which were then being considered by the Building Department of New York City. The speaker was requested to write a letter to the Superintendent of Buildings if his examination was satisfactory. He calculated the strength of the floors as closely as he could with the formulas at his disposal, and found that they had only about one-twelfth of the requisite strength. Some time later, upon inquiry he found that an experimental arch had been built and tested; that it had stood the test, and that the plans had been accepted.

The materials in such arches are so radically different that in computing their relations to one another as to their moduli there may be considerable error.

The author's formulas can only be proved to be correct by experiment. With an iron rod at the bottom of the concrete beam, instead of the expanded metal, the conditions are very much the same as they would be with an iron rod at the bottom of a wooden beam; and in the latter, which is understood a little better than the concrete, there is a certain amount of the wood fiber in compression in the bottom part. There is also a certain part of the fiber in tension at the same time that the steel is in tension. This passes practically out of tension when the elastic properties of the wood step in to stop taking up

any more stress, and dependence has to be placed altogether on the strength of the steel, which has perhaps thirty times the stiffness or lack of elasticity. This is also the case with expanded metal. Actual results must be obtained by numerous experiments. Meanwhile, the author's formulas, which are correct in form, and which agree with the results of experiments, may be used. Mr. O'Rourke.

In giving the results of this series of experiments, so different from any theories on the subject, the author has conferred a benefit on the profession, and should be encouraged to continue his experiments.

GUY B. WAITE, Assoc. M. Am. Soc. C. E.—The data produced by the author, through these elaborate experiments on composite fire-proof floor construction, is extremely interesting at a time when so much attention is being given to the various combinations of concrete and iron. Mr. Waite.

From the painstaking and conscientious records of the materials used, their manipulation, and their behavior at the various stages of the tests, one can form a good judgment of the characteristics and of the proper uses of this and of similar forms of construction.

Passing over the consideration of the theoretical part of the paper to the more practical part, one is impressed with the great variations shown in the maximum strengths of the various pieces of similar materials tested. This is especially noticeable with the cinder concretes in Table No. 3, where, under similar conditions, the maximum load borne by some pieces is about three times that borne by others.

It is not surprising that cinder concrete should show a wide variation in strength, as homogeneousness, which is necessary to form a uniformly substantial base for making a strong concrete, is lacking in the cinders used. It is somewhat surprising, however, to note that the best cinder concretes withstood loads nearly as great as the best stone concretes.

It is believed that these tests show conclusively that in practict there is too wide a variation and too much uncertainty in the action of the composite construction to warrant its use with practically the same uniformity and for the same purposes as steel beams are now used.

The introduction of construction of this character for long spans, as proposed by the author in Table No. 2, is objected to, both on the ground of questionable construction and of economy in the weight and cost of the necessary materials as set forth in the paper.

The working strains assumed by the author seem conservative for good concretes, but they are based on the tests submitted and on a factor of safety of only 5. In actual construction, without the constant supervision of a competent person, it is doubtful whether more uniform results than those of the author could be obtained.

Taking an average of those deflections submitted which are within the elastic limit, and using the modulus of elasticity thus obtained, the longer spans given in Table No. 2 will have deflections greatly

Mr. Waite. exceeding that allowed in good construction, unless it can be shown clearly that in every case the construction will positively be made fixed at each end—or truly continuous over supports. When steel beams were sold for about twice their present price, there was, perhaps, economy in using the material composing the floor construction as a compression member, as in the "Ransome" and the "Lee" systems of construction, where steel tension members are united respectively with concrete and with burned clay, floor-compression members, but with current prices for steel beams there is no such advantage, as will presently be shown. Taking the author's safe working value for compression on stone concrete at 150 lbs. per square inch, and assuming the allowed working compression on the extreme fiber of steel beams at 16 000 lbs. per square inch the comparative values of equal sections of the two kinds of materials will be as 1 : 107.

Assuming that stone concrete, such as the author used for the experiments, can be furnished and put in place in the floor complete (irrespective of temporary centers) for 20 cents per cubic foot, and that the steel beams are to cost 2 cents per pound,* the relative costs of the two materials, considering equal quantities, will be as $1 : 48\frac{8}{10}$. Therefore, with the above assumptions, the actual comparative economical values of the steel beams and the concrete for compression members will be as $48\frac{8}{10} : 107$, or, in other words, the steel beam will do about $2\frac{1}{4}$ times the work of the concrete at a given cost without considering the more economical cross-section of the I-shaped steel beam. For comparing the economy in the moments of inertia of the steel beam and of the solid concrete, take, for example, an 8-in. 18-lb. per foot I-beam, having a sectional area of 5.4 sq. ins., and a moment of inertia of 57.8, and compare it with 5.4 sq. ins. of solid concrete 8 ins. deep, having a moment of inertia of 28.8, and it will be seen that the economy of the sectional areas is as 1 : 2.

Therefore, everything considered, the real theoretical comparison of the economical values of concrete and steel I-beams is approximately $2\frac{1}{8} \times 2 = 4\frac{2}{8}$; that is, the steel beam will carry in compression the same load as a given concrete beam at one-quarter of the cost. With cinder concrete the advantages of the steel will be still more apparent.

For the tensional part of the materials considered, which, in the composite construction, is taken up by expanded metal, twisted wire, rods, or cables, it seems hardly necessary to prove that, at the prices above assumed, rolled steel beams can be used much more economically for taking tensional stresses than any of the other materials referred to.

For the sake of giving the most liberal comparison, however, let it be assumed that the tensional part of the two classes of construction can be used at the same cost and let the comparison for economy be based entirely on the compression. Then the cost of steel beams

* At present they can be furnished and placed in the floor, complete, for considerably less than this price.

would be one-quarter of the theoretical cost of concrete, each having the same depth when both materials are engaged to their working limits in a floor construction. Mr. Waite.

It will be contended that a floor construction of some kind must be used between steel beams in any case, and that it will be economical to harness it up and use it in the way the author prescribes; that is really the practical destination to which this argument is intended to be directed, to show that there are practical and economical limitations in the manner of using the author's construction. To compare the practical values of this construction, having the long spans shown in Table No. 2, with the steel-beam construction, having ordinary spacings filled with the author's or with other concrete construction, take the case shown in the table, having 175 lbs. per square foot live load and 15 ft. span.

With this span the concrete is 10 ins. thick, but with a 5-ft. span the concrete is 5 ins. thick, and as the tensional part of the author's construction is assumed at the same cost as the lower or tensional half of the steel beams, as heretofore set forth, the following comparison of the relative costs of the author's construction, and of the steel I-beam construction, for the 15-ft. span may be made.

A 10-in. I-beam, 18 lbs. per foot, when spaced at 5-ft. centers over a clear span of 15 ft., will carry 232 lbs. per square foot, and, according to the table, the thickness of concrete necessary for this span is only 5 ins. Hence, by using the 5-ft. spaced I-beam construction, rather than the 15-ft. span concrete construction, the saving in concrete will be $\frac{5}{12} \times 5 \times 15$ ft. $= 31\frac{1}{4}$ cu. ft., which, at 20 cents per cubic foot, $= \$6.25$.

Assuming that a 10-in. I-beam, 25 lbs. per square foot, is used, the weight of the steel for the clear span considered will be 375 lbs.

The extra cost of one-half of this weight (the compression part), or 188 lbs., is to be compared with the saving in concrete above determined. This 188 lbs. of steel, at the rate previously assumed, 2 cents per pound, will cost \$3.75.

Therefore, it is seen that, with the longest spans shown in Table No. 2, the author's construction would not be economical, when compared with steel beams and short arches. The other work connected with the author's construction, which has not here been considered, is about the same, whether the concrete is 5 ins. or 10 ins. thick.

Theoretically, there will be a certain spacing of the floor beams, connected with different loadings, which will give a thickness of concrete between the beams of minimum cost to both steel beams and to concrete combined.

Hence, by using the 5-ft. spaced I-beam construction, rather than the 15-ft. span concrete construction, the saving in concrete will be $\frac{5}{12} \times 5 \times 15$ ft. $= 31\frac{1}{4}$ cu. ft., which, at 20 cents per cubic foot, $= \$6.25$.

CORRESPONDENCE.

J. B. Johnson. J. B. JOHNSON, M. Am. Soc. C. E.—The writer has tried to read the theoretical part of this paper, beginning on page 628, but finds difficulties in doing so. The subject is a new one and is as important as it is new, and hence of great practical interest to engineers. The writer knows of but two published analyses for the strength of a combined steel and concrete beam.* While these two investigations proceeded on entirely distinct lines, the resulting equations are identical, and he thinks quite correct, inside the elastic strength of the concrete, which is practically its ultimate strength in tension. In other words, these analyses give the strength of steel-concrete beams up to the time the concrete cracks on the tension side. For the ultimate strength of such beams, which is some three times that developed when the concrete cracks, the writer gave an analysis at the meeting of the American Association for the Advancement of Science, in August, 1897.† In none of these, however, was the theory applied to the use of expanded metal, and such an application is especially difficult, because of the extreme variations in the cinder concrete, commonly used for floors, and also as to the modulus of elasticity of expanded metal when embedded in such concrete. It is probable that this could only be determined by making tests of slabs of cinder concrete with and without expanded metal embedded therein. It would seem, therefore, that the data is not yet at hand to enable working formulas to be derived for the designing of such beams as were here tested, at least for evaluating the constants in such a formula. A theoretical formula might, however, be derived, as those cited above, and tests made to determine the values of the constants used.

The author seems to have become confused as to resisting stresses and the moments of those stresses. Thus he says (page 629) that the "compression is represented by $\frac{Cx^2}{3}$." He has just stated that "the compression in the concrete above the neutral axis must equal the tension below that axis," which is correct (the author giving an entirely erroneous reason for this equality, however), and he here uses the terms compression and tension correctly. When he next uses them, however, he appears to regard them as moments, since $\frac{Cx^2}{3}$ is the moment of the compressive stress about the neutral axis.

The assumption is also made that the steel reaches its working stress simultaneously with that of the concrete in tension. They do, in fact, deform equally, and, because of this equal deformation, their

*J. B. Johnson in *Engineering News*, January 3d, 1895; elaborated in "Materials of Construction," by J. B. Johnson. Also "Baumaterialen-kunde," by Julius Mandl.

† Published in *Engineering News* of October 21st, 1897.

stresses are as their relative moduli of elasticity. While the modulus of elasticity of cinder concrete is unknown, it is certainly quite variable and very low; it is less than 1 000 000 and possibly may be as low as 100 000. This means that the tensile stress in the expanded metal will be from 30 to 300 times as much per square inch as that in the concrete, but these relative stresses cannot be taken at pleasure on the assumption that they will come to their respective working limits at the same time, as is done in this paper. J. B. Johnson.

Formula (2) is based on the assumption that the moments of the stresses on the two sides of the neutral axis are equal to each other, which is never the case in a composite steel-concrete beam. No reason is assigned for the correctness of formula (3), and there is no evident rational determination for it.

The writer cannot but regard the theoretical part of this paper, therefore, as erroneous and misleading.

A. L. Johnson, Assoc. M. Am. Soc. C. E.—As engineer of the St. Louis Expanded Metal Fireproofing Company, the writer is much interested in the subject of this paper. Though asked by associates in the East to participate in these tests, the writer's company was not represented. The results obtained were far from satisfactory, being seldom more than one-third of the true strength as obtained by tests of the same age and mixture of concrete, upon panels of a floor in a building. A. L. Johnson.

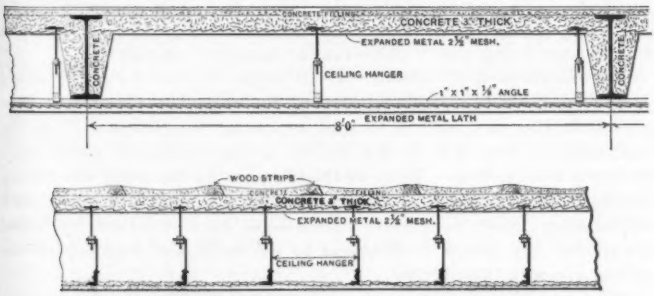


Fig. 14.

The St. Louis Expanded Metal Fireproofing Company's tests, made during the last three years, show for a flat arch (Fig. 14), composed of cinder concrete, mixed in the proportions of one of cement, two of sand and five of cinders, and using No. 16 gauge, 2½-in. mesh, expanded metal, on a span of 6 ft. 9 ins., a strength averaging 700 lbs. per square foot, uniformly distributed over the whole surface. As opposed to this, the tests reported in the paper show for a 6-ft. span,

A. L. Johnson.

$4\frac{1}{2}$ ins. thick, with metal of No. 10 gauge, 3-in. mesh (the latter being 20% stronger than the metal used in the above test), and of the same mixture of concrete (see test No. 61), a strength of only 330 lbs. per square foot, a result certainly very disappointing to the manufacturers, especially as it is so much less than the real value of the construction.

There are, probably, three elements which contribute to this disastrous result. These are:

1st. The incorrect location of the expanded metal in the slab.

2d. The failure to test the construction as actually employed in a building.

3d. The conditions of the weather between the time of mixing the concrete and the testing of the panels, during a large portion of which time the thermometer was from 2° to 11° below the freezing point.

With regard to the first element, it is highly important that the expanded metal should be placed as low as possible in the concrete beam, so as to develop the tensile strength of the steel used. In the construction of this work it is customary to lay the expanded metal directly upon the centering before doing any concreting, excepting in the haunches along the beams, as the metal is so constructed that when lying on the centering, or any flat surface, it touches only at the points where the strands unite. It is practicable to do this, and yet have the metal entirely covered by the concrete after it is put in position. The increase of strength due to the metal is directly proportional to its distance from the neutral axis of the concrete beam, the increase being zero if placed in the middle of the slab.

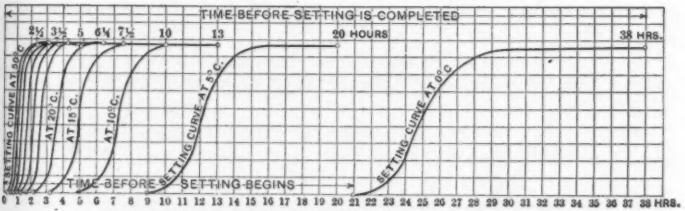
An examination of column "d" of Table No. 3 of the paper would seem to indicate that in a majority of the cases the metal had been placed in this way, as $\frac{1}{4}$ in. would represent the distance from the underside of the slab to the middle of the expanded metal when properly constructed. Many of the tests are so far from developing the theoretical strength for such construction, this theoretical strength conforming closely to results which have been obtained by actual tests, that the writer is disposed to discredit the accuracy of the values given in this column.

The second element is also of great importance. As actually placed the construction would have haunches built down upon the webs of the beams, and the majority of the construction would have panels on either side. Now, while the writer has developed a formula (to follow herein) which, as he has said, checks with the practical results obtained from tests on slabs of this combined construction when simply supported at the ends, the actual test on a middle panel in a floor, will give results on the average about twice as great as obtained by this formula or as obtained from tests on isolated slabs supported at the ends. This is due to the development in the floor

panel of a segmental arching action within the thickness of the slab itself, the floor on each side taking up the thrust. The wall panels, of course, would not develop as great a strength as this unless the walls were quite heavy, but these panels might be made of different construction to provide for this.

With regard to the third element, the state of the weather, it will be seen from an examination of the dates (none of which are given in the paper, but are given in a monograph report published by the Expanded Metal Companies) that much of the concreting was done on days when freezing occurred. Even if it was not freezing at the time the concrete was being mixed, it would nevertheless be affected by the low temperature to the extent of very materially delaying the time of setting, as well as its rate of increase in strength after setting. This is shown by the diagram, Fig. 15.*

The material was certainly extremely irregular in its quality, which is best shown by an inspection of the results obtained from the compression tests included in Table No. 3.


TIME BEFORE SETTING IS COMPLETED
2½ 3½ 5 6¼ 7½ 10 13 20 HOURS 38 HRS.
SETTING CURVE AT 40°C.
AT 20°C.
AT 15°C.
AT 10°C.
SETTING CURVE AT 5°C.
SETTING CURVE AT 0°C.
TIME BEFORE SETTING BEGINS
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 HRS.

GRAPHICAL REPRESENTATION OF THE RATE OF SETTING OF PORTLAND CEMENT AT VARIOUS TEMPERATURES

FIG. 15.

For example, test No. 5, stone concrete mixed 1 : 3 : 6, gave a compressive strength of 398 lbs. per square inch, while test No. 7, combined stone and gravel, mixed 1 : 2 : 10, gave a compressive strength of 625 lbs. per square inch; that is to say, a considerably weaker mixture showed about 60% more strength.

Test No. 23, cinder concrete, 1 : 2 : 5 gave 305 lbs. per square inch, while test No. 27 of the same mixture gave 490 lbs. per square inch. Test No. 48, cinder concrete mixed 1 : 1.6 : 4 showed a strength of 330 lbs. per square inch, while tests Nos. 50 and 51 on exactly the same mixture gave a strength of 765 lbs. per square inch, or about $2\frac{1}{3}$ times as much.

Test No. 1, cinder concrete mixed 1 : 3 : 6 gave 292 lbs. per square inch, while test No. 32, 1 : 2.4 : 6, or practically the same mixture, gave 590 lbs. per square inch.

* See Johnson's "Materials of Construction."

A. L. Johnson.

In Fig. 16, the writer presents a diagram which he has had for about two years, giving the strength of various mixtures of cinder concrete, the broken lines being the strength at 28 days and the full lines the estimated strength at the end of six months. Upon this diagram he has plotted the tests made by the author.

The diagram contains all the writer's experiments, which are indicated by small circles, and each of which is the mean of two results. The results obtained by the author are indicated by crosses.

While the writer's tests grow uniformly weaker, as the proportion of cement decreases, as should be the case, it will be seen that the results given in the paper exhibit a very great degree of irregularity in this regard, and are also materially below the results obtained by the writer. The author's results certainly indicate extremely poor

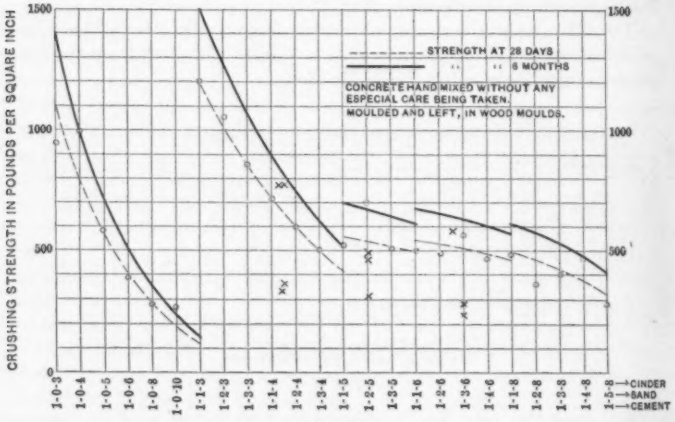


FIG. 16.

handling in some way. Concrete, with the most ordinary kind of treatment, ought not to vary more than 20% in strength, and, with the care usually demanded and exercised, will give results in which the extreme values will not differ more than 10%, while some of the author's vary considerably more than 100 per cent.

A discussion of the theory of the moment of resistance of this class of construction, evolved by the writer about a year ago, which is presented herewith, is offered as a substitute for the equations derived by the author, and which are regarded as very far from expressing the true conditions.

This discussion was developed for the Ransome system of floor construction, upon the hypothesis that the bond between the concrete and the twisted rods remained unbroken. This is true when the floor-

ing is first built, but in many cases it is ultimately broken by vibrations and shocks. The formulas, however, are applicable to the expanded metal construction for all time, as, owing to the shape of the metal, dependence is not placed upon this adhesion. To adapt the formulas, therefore, to the expanded metal construction, it is only necessary to properly evaluate the quantity $\frac{2a^2}{d}$

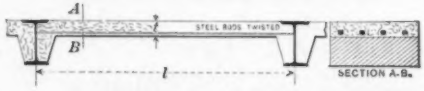


FIG. 17.

Let E_c = modulus of elasticity of concrete, in pounds per square inch.

E_s = modulus of elasticity of steel, in pounds per square inch.

f = cross breaking modulus of rupture for concrete beams.

$*F$ = the elastic limit of steel, in pounds per square inch.

C = the compressive strength of concrete, in pounds per square inch.

t = the thickness of the concrete slab, in inches.

a = width of square steel rod, in inches (for a rod 1 in. square, $a = 1$ in.).

d = distance between rods, in inches.

e = distance from center line of rods to middle line of beam or slab, in inches.

r = distance from middle line of slab to the neutral axis of the combined beam construction, in inches.

b = breadth of section, in inches.

l = the length of span, in feet.

w = load, in pounds per square foot.

m = width of wings produced by replacing the area occupied by the rods with its equivalent area in concrete,† as obtained from their relative moduli of elasticity.

I = the moment of inertia of the cross-section.

M_o = the moment of resistance of the cross-section in inch-pounds.

M = the bending moment of the external forces in inch-pounds.

For stone concrete, E_c being about 1 000 000,

$$m = \frac{b}{d} \times a \times \frac{E_s}{E_c} = \frac{30\,a\,b}{d}.$$

* For this purpose the elastic limit is taken at 40 000 lbs. per square inch, a little beyond the true elastic limit, as, when used in combination with concrete, the steel would probably be stressed somewhat beyond the limit.

† For this method of analysis see an article by Professor J. B. Johnson in *Engineering News*, January 3d, 1895, and also his "Materials of Construction."

A. L. Johnson.

To determine the position of the neutral axis, take moments about the center line of the slab, Fig. 18. Dividing the moment of the entire area about this axis by that area, the distance to the center of gravity or to the neutral axis is found to be:

$$r = \frac{a\,m\,e}{b\,t + a\,m}$$

Or, substituting for m its value as given above,

$$r = \frac{a^2\,e\,E_s}{d\,t\,E_c + a^2\,E_s} = \frac{e}{1+k}\,;\ \text{where } k = \frac{t\,d}{a^2} \times \frac{E_c}{E_s}.$$

Also, the moment of inertia of the total area is

$$I = \frac{b\,t^3}{12} + b\,t\,r^2 + \frac{m\,a^3}{12} + a\,m\,(e-r)^2 = \frac{b\,t^3}{12} + \frac{m\,a^3}{12} + \frac{b\,t\,e^2}{1+k}.$$

Example 1.—Let $t = 6$ ins., $b = 12$ ins., $a = 1$ in., $d = 6$ ins., $E_c = 1\,000\,000$, $E_s = 30\,000\,000$; $e = 2.25$ ins.; which is an average case of actual practice, except that the rods are larger than could generally be used.

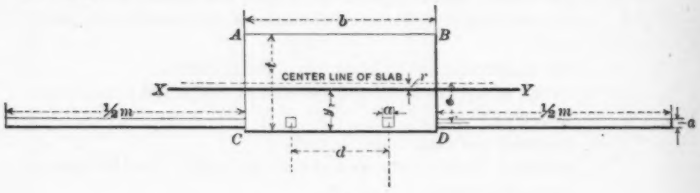


FIG. 18.

Then, for the second member of the above equation,

$$\frac{b\,t^3}{12} = 216.0$$

$$\frac{m\,a^3}{12} = 5.0$$

$$\frac{b\,t\,e^2}{1+k} = 165.6$$

Total moment of inertia, $I = \overline{386.6}$

The second term of this equation gives a value of only 1½ % of the total value of I, and it is much larger for the case assumed than it would ordinarily be; the percentage of error, therefore, will be immaterial if this member is entirely omitted,

making $$I = b\,t\left(\frac{t^2}{12} + \frac{e^2}{1+k}\right).$$

Case I. Concrete not to Crack.—The above value of I is the proper one to use for the determination of the load-carrying capacity of the

combination, up to the time the concrete cracks on the tension side. A. L. Johnson.
To determine how much this will be:

$$M_o = \frac{f\,I}{y_2};\ \text{where } y_2 = \frac{t}{2} - r.$$

Example 2.—Taking f at 300 lbs. per square inch and the other values as assumed in the previous example:

$$M_o = 57\,800 \text{ in.-lbs.} = 4\,817 \text{ ft.-lbs.}$$

Also, $$M = \frac{w\,l^2}{8} = M_o$$

or, $$\frac{w\,l^2}{8} = 4\,817\,;\ \text{from which } w\,l^2 = 38\,536\,;$$

or, if $l = 10$ ft., $w = 385$ lbs. per square foot, as the load that can be carried, up to the time that the concrete will crack in tension on the under side.

Case II. Ultimate Strength.—Suppose the slab of concrete has cracked on the tension side, but the loading is still continued; the neutral axis then rises until the total compression above the axis is equal to the total tension on the projecting wings alone. The crack

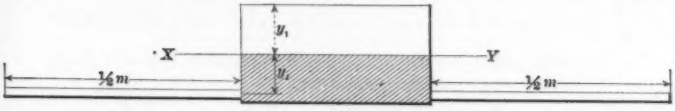


FIG. 19.

will not extend quite as high as the neutral axis, but so near it that the slight moment of inertia due to this small area immediately below the neutral axis may be neglected.

The hatched portion of Fig. 19 represents the area no longer serviceable in resisting bending.

To determine the new position of the neutral axis:

$$y_1^2 = \frac{2\,a^2}{d} \times \frac{E_s}{E_c} \times y_2, \qquad \text{or } y_2 = \frac{y_1^2\,d\,E_c}{2\,a^2\,E_s}$$

Also, in practice,

$y_1 = t - y_2 - \frac{a}{2} - 0.25$ in.; since the bars are imbedded so that their bottom portion is about $\frac{1}{4}$ in. from the under side of the slab; the object being to get them as far down on the tension side as possible, without leaving them exposed.

Example 3.—Assuming quantities as in the previous case:

$$y_1^2 = 10\,y_2$$

also, $$y_1 = 5\tfrac{1}{4} \text{ in.} - y_2\,;\ \text{from which}$$

$$y_1^2 + 10\,y_1 + 25 = \frac{210}{4} + 25$$

A. L. Johnson.

or, $y_1 + 5 = \pm \sqrt{77.5}$, from which is obtained

$y_1 = 3.8$ ins.

and $y_2 = 1.45$ ins.

The moment of inertia of the new cross-section is

$$I = \frac{b\, y_1^3}{3} + m\, a\, y_2^2$$

The moment of resistance is $M_o = \frac{f I}{y_2}$, where, however, f has a peculiar value, determined as follows: The area $m\, a$ is here shown as 30 times the original area of the cross-section occupied by the bars, which it should be for the determination of the proper moment of inertia; but f, for concrete, = about 300 lbs. per square inch, while F, for steel, = about 40 000 lbs. per square inch; that is to say, for steel bars in stone concrete,

$$E_s = 30\, E_c$$
$$F = 133\, f_c$$

Hence, the loading may be continued 4.4 times as far as if the material in the wings were really concrete, though the deflections for each increment will be the same as if it were all of the latter material.

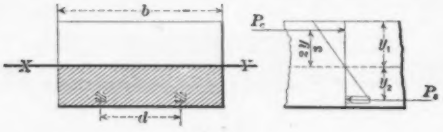


FIG. 20.

Then take for the new value,

$$f = f_c \left(\frac{E_c}{E_s} \times \frac{F}{f_c} \right) = \frac{F\, E_c}{E_s}.$$

Then

$$M_o = \frac{F\, I\, E_c}{y_2\, E_s}; \text{ and by substituting for } I, \text{ its value,}$$

there is obtained:

$$M_o = \frac{F\left(\frac{b\, y_1^3}{3} + m\, a\, y_2^2\right) E_c}{y_2\, E_s}$$

But

$$y_2 = \frac{y_1^2\, d\, E_c}{2\, a^2\, E_s} \text{ and } m = \frac{a\, b}{d} \times \frac{E_s}{E_c}.$$

From which, by substituting and reducing, is obtained:

$$M_o = F\, b \left(\frac{2\, a^2\, y_1}{3\, d} + \frac{E_c}{E_s} \times \frac{y_1^2}{2} \right).$$

Another Discussion of Case 2.—For this case, assume that the loading has proceeded until the steel rods have been stretched to their

elastic limit; the total stress in the rods is equal to the total amount of compression in the concrete above the neutral axis. A. L. Johnson.

Also, the actual elongation of the rods, per unit of length, is to the compression of the extreme upper fiber, per unit of length, as y_2 is to y_1.

This relation is simply geometrical, and is only dependent upon the moduli of elasticity for both the steel and concrete remaining constant up to this point; as a matter of fact the modulus of elasticity of the steel is practically constant up to its elastic limit. This is not true, however, of the modulus of elasticity of the concrete, and the modulus used here should be determined by the tangent of the angle of a line on the compressive stress diagram* for the given class of material, drawn from the origin of coördinates to the point on the curve corresponding with the compression per square inch, developed in the extreme fiber of the beam under consideration.

The total area of the steel rods $= \frac{a^2 b}{d}$.

Then, total stress on rods $= P_s = \frac{F a^2 b}{d}$.

The stretch of the rods, or strain, per unit of length, $\lambda_2 = \frac{F}{E_s}$.

And, where A is equal to the cross-sectional area of the rods,

$$P_s = E_s \lambda_2 A.$$

Now, the compression of the extreme fiber on the other side of the neutral axis is

$$\lambda_1 = \lambda_2 \frac{y_1}{y_2};$$

and the intensity of the stress in the extreme fiber is

$$f_c = \lambda_1 E_c = \lambda_2 \frac{y_1}{y_2} E_c.$$

Also,

$$P_c = f_c \frac{b}{2} \times y_1 = \frac{b \lambda_2 {y_1}^2 \times E_c}{2 y_2} = \frac{F b {y_1}^2 E_c}{2 y_2 E_s}.$$

But,
$$P_c = P_s$$

Hence,

$$E_s \lambda_2 A = \frac{b \lambda_2 {y_1}^2 E_c}{2 y_2}.$$

Then,

$$2A y_2 E_s = b {y_1}^2 E_c;$$

from which is obtained:

$$y_1^2 = \frac{2 a^2}{d} \frac{E_s}{E_c} y_2.$$

* Such compressive stress diagrams for concrete may be found in Johnson's "Materials of Construction."

A. L. Johnson.

When the rods are stretched, up to their elastic limit, the moment of resistance, neglecting the resistance due to bending the rods, and the slight moment before mentioned due to the small area of concrete still intact immediately below the neutral axis, is

$$M_o = P_s y_2 + \frac{2 P_c y_1}{3} = P_s \left(y_2 + \frac{2 y_1}{3} \right)$$
$$= \frac{F a^2 b}{d} \left(y_2 + \frac{2 y_1}{3} \right) = \frac{F a^2 b}{d} \left(\frac{y_1^2 d E_c}{2 a^2 E_s} + \frac{2 y_1}{3} \right)$$
$$= F b \left(\frac{E_c}{E_s} \frac{y_1^2}{2} + \frac{2 a^2 y_1}{3 d} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This is exactly the same equation as obtained from the first discussion.

Let, as before, $t = 6$ ins.; $b = 12$ ins.; $d = 6$ ins.; $E_s = 30\,000\,000$;
$E_c = 1\,000\,000$; $a = 1$ in.; $F = 40\,000$.

From which, as before, $y_1 = 3.8$ ins. and $y_2 = 1.45$ ins.; then

$$M_o = 480\,000 \left(\frac{1}{30} \times 7.2 + \frac{2}{18} \times 3.8 \right)$$
$$= 480\,000 \ (\ .662\ ) = 320\,000 \text{ in.-lbs.}$$

For the case assumed, it is seen that, up to the time that the concrete cracks on the under side, only about one-fourth of the ultimate strength has been developed.

If the slab is thin the concrete in its top may fail in compression. The moment of resistance for this case is:

$$M^1{}_o = P_s y_2 + \frac{2 P_c y_1}{3} = P_c y_2 + \frac{2 P_c y_1}{3}$$

but,

$$P_c = \frac{b f_c y_1}{2}$$

Hence,

$$M^1{}_o = \frac{b f_c y_1 y_2}{2} + \frac{2 b f_c y_1^2}{6} = f_c b \left(\frac{y_1 y_2}{2} + \frac{y_1}{3} \right)$$
$$= \frac{f_c b y_1^2}{2} \left(\frac{y_2}{y_1} + \frac{2}{3} \right)$$

But,

$$y_2 = y_1^2 \times \frac{d}{2 a^2} \times \frac{E_c}{E_s}$$

Hence,

$$M_o^1 = \frac{f_c b y_1^2}{2} \left(\frac{d}{2 a^2} \times \frac{E_c}{E_s} \times y_1 + \frac{2}{3} \right) \ldots\ldots\ldots\ldots\ldots (2)$$

Designing.—To design a floor, on the Ransome system, to carry a given load on a given length of span, it is necessary first to decide on the factor of safety to be used.

It is not desirable that the concrete should crack underneath at the full working load, so that there should be some margin of safety at this point. Suppose the factor of safety is made 1.5 at this point. It certainly should not be any less. This will give a factor of safety

at the point of failure of about 6; as the cracking point, as has been seen, is at about one-fourth the ultimate load.

A. L. Johnson.

Let $w = 150$ lbs. per square foot;

$l = 10$ ft.

Then,

$$M = \frac{12\ w\ l^2}{8} = M_o = \frac{F\ b}{6}\left(\frac{E_c}{E_s}\ \frac{y_1^2}{2} + \frac{2\ a^2\ y_1}{3d}\right).$$

In this equation w and l, F, E_c and E_s are known; b is assumed at 12 ins., and d and a are to be given trial values.

Let $d = 3$ ins. and $a = \frac{1}{2}$ in.

Then,

$$\frac{480\,000}{6}\left(\frac{1}{30}\ \frac{y_1^2}{2} + \frac{y_1}{18}\right) = \frac{180\,000}{8}.$$

Or,

$$\frac{y_1^2}{60} + \frac{y_1}{18} = 0.28125.$$

From which is obtained: $y_1 = 2.76$ ins.

$$y_2 = \frac{y_1^2\ d\ E_c}{2\ a^2\ E_s} = \frac{7.62 \times 3 \times 1\,000\,000}{2 \times \frac{1}{4} \times 30\,000\,000} = 1.52 \text{ ins.},$$

and hence,

$$t = y_1 + y_2 + \frac{a}{2} + \tfrac{1}{4} \text{ in.} = 4.78 \text{ ins.}$$

That is to say, a slab $4\frac{3}{4}$ ins. thick, with $\frac{1}{2}$-in. twisted rods spaced 3 ins. apart on the bottom side of the same, will carry with safety 150 lbs. per square foot on a 10-ft. span. This, of course, includes the dead load, which would have to be deducted to give the live load carrying capacity.

An economical combination for this class of construction would be such as would render the beam equally liable to fail, either by the crushing of the concrete on top or by the rupture of the metal on the bottom.

To determine how much metal should be used, there exists the condition that the two moments of resistance for these two methods of failure are equal, or,

$$\frac{F\ I}{y_2} \times \frac{E_c}{E_s} = \frac{f_c\ I}{y_1},$$

where f_c is the modulus of rupture of the concrete in compression. For rock concrete, f_c is about 1 500 lbs. per square inch.

Here the values y_1 and y_2 are dependent upon the amount of metal used in the construction.

From the above is obtained:

$$\frac{F}{y_2} \times \frac{E_c}{E_s} \quad \frac{f_c}{y_1}$$

Or,

$$y_2 = \frac{F\ E_c}{f_c\ E_s} \times y_1.$$

A. L. Johnson.

But,
$$y_2 = \frac{y_1^2\, d\, E_c}{2\, a^2\, E_s},$$

From which
$$y_1 = \frac{F}{f_c} \times \frac{2\, a^2}{d}.$$

Or,
$$\frac{a^2}{d} = \frac{f_c y_1}{2\, F}.$$

Expanded Metal Slabs.—Take a sheet of No. 16 gauge, $2\frac{1}{2}$ mesh, expanded metal. Here
$$\frac{a^2}{d} = 0.01224.*$$

Then $y_1 = 0.0245\, \frac{F}{f_c}$, where, for cinder concrete, $f_c = 600$ lbs. per square inch.

Then
$$y_1 = 1.63 \text{ ins.},$$
$$y_2 = \frac{y_1^2\, d\, E_c}{2a^2\, E_s}.$$

Here, on account of the diamond shape of the meshes, E_s should be divided by the secant of one-half the angle at the point of the diamond = 1.11.

This makes $E_s = 25\,700\,000$.

Then
$$y_2 = \frac{1.63^2 \times 40.8 \times 550\,000}{25\,700\,000} = 2.33 \text{ ins.}$$

Then $t = y_1 + y_2 + \frac{1}{4}$ in. $= 4.21$ ins.

That is to say, that No. 16 gauge, $2\frac{1}{2}$-in. mesh, expanded metal will develop the compressive strength of a cinder concrete slab $4\frac{1}{4}$ ins. thick, supposing that the compressive strength of the same does not exceed 600 lbs. per square inch, and that there has been no loss in the gross area of the expanded metal sheet due to the cutting. There is always some loss of this kind, however, so that the writer is accustomed to take $\frac{a^2}{d} = 0.01$ to allow for it. This value will give $y_1 = 1.33$ ins., $y_2 = 1.89$ ins. and for t, 3.47 ins., or about $3\frac{1}{2}$ ins., as the maximum thickness of a cinder concrete slab which No. 16 gauge, $2\frac{1}{2}$-in. mesh metal will develop, a condition which corresponds with the actual tests made.

In computing the carrying capacity of expanded metal slabs simply supported at the ends and not as used in a floor, the formulas marked 1 and 2 may be used.

Formula 1 is used if the slab is destined to break in tension, and formula 2 if in compression.

---

* No. 16 gauge metal weighs $2\frac{1}{2}$ lbs. per square foot, and is, therefore, 0.0612 in. thick. Then $\frac{a^2}{d} = 0.0612 \times 8$ ins. $\times \frac{1}{40} = 0.01224$ sq. in. of metal per inch of width of sheet, the sheets being expanded from an original width of 8 ins. to a final width of 40 ins.

The value $\frac{2\,a^2}{d}$, of course, depends upon the style and quantity of expanded metal used.

A. L. Johnson.

TABLE No. 4.

No. of Test.	Foundation material.	Cement.	Proportions used.	Thickness.	Span.	$\frac{2\,a^2}{d}$	$\frac{E_s}{E_c}$	M_o	$\frac{12\,l^2}{8}$ ("l" in feet).	W = computed carrying capacity in lbs. per square foot uniform distribution.	W' result of test.
1..	C	Atlas	1 : 3 : 6	3.87	5′	$\frac{1}{16}$	57.0	18 900	37.5	504	287
3..	C	"	1 : 2 : 5	3.75	5′	$\frac{1}{16}$	51.4	18 200	37·5	485	393
4..	C	"	1 : 2 : 5	4.00	5′	$\frac{1}{16}$	51.4	19 700	37.5	525	478
5..	S	"	1 : 3 : 6	3.50	5′	$\frac{1}{16}$	12.8	19 900	37.5	530	297
6..	S	"	1 : 3 : 6	3.50	5′	$\frac{1}{16}$	12.8	19 900	37.5	530	325
7..	S	"	1 : 2 : 10	3.74	5′	$\frac{1}{16}$	17.0	20 300	37.5	541	345
8..	S	"	1 : 2 : 10	3.74	5′	$\frac{1}{16}$	17.0	19 100	37.5	510	287
9..	G	"	1 : 3 : 6	3.62	5′	$\frac{1}{16}$	8.6	20 800	37.5	555	402
10..	G	"	1 : 3 : 6	3.75	5′	$\frac{1}{16}$	8.6	21 500	37.5	573	440
11..	G	"	1 : 2 : 7	3.81	5′	$\frac{1}{16}$	8.6	22 500	37.5	600	580
12..	G	"	1 : 2 : 7	4.50	5′	$\frac{1}{16}$	8.6	26 400	37.5	703	422
13..	C	Silica	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	3.81	5′	$\frac{1}{16}$	51.4	20 100	37.5	535	440
14..	C	"	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	3.75	5′	$\frac{1}{16}$	51.4	19 700	37.5	525	502
15..	C	"	1 : $1\frac{6}{10}$: $4\frac{6}{10}$	3.50	5′	$\frac{1}{16}$	51.4	18 300	37.5	488	407
16..	C	"	1 : $1\frac{6}{10}$: $4\frac{6}{10}$	3.50	5′	$\frac{1}{16}$	51.4	18 300	37.5	488	484
17..	S	"	1 : 3 : 5	3.75	5′	$\frac{1}{16}$	12.8	21 500	38.5	572	570
18..	S	"	1 : $2\frac{4}{10}$: 4	3.50	5′	$\frac{1}{16}$	12.8	19 900	37.5	530	431
19..	G	"	1 : 3 : 6	3.81	5′	$\frac{1}{16}$	8.6	22 500	37.5	600	498
20..	G	"	1 : 3 : 6	4.06	5′	$\frac{1}{16}$	8.6	23 600	37.5	630	575
21..	G	"	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	4.06	5′	$\frac{1}{16}$	8.6	23 600	37.5	630	535
22..	G	"	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	3.38	5′	$\frac{1}{16}$	8.6	19 000	37.5	506	431
23..	G	"	1 : $1\frac{6}{10}$: $6\frac{4}{10}$	3.81	5′	$\frac{1}{16}$	8.6	22 500	37.5	600	469
24..	G	"	1 : $1\frac{6}{10}$: $6\frac{4}{10}$	3.94	5′	$\frac{1}{16}$	8.6	23 000	37.5	612	545
33..	C	Acme	1 : 3 : 9	3.62	5′	$\frac{1}{16}$	64.2	17 700	37.5	472	159
34..	C	"	1 : 3 : 6	4.12	5′	$\frac{1}{16}$	57.0	21 800	37.5	581	645
35..	C	"	1 : 2 : 5	3.44	5′	$\frac{1}{16}$	51.4	17 700	37.5	472	684
36..	C	"	1 : 2 : 3	3.88	5′	$\frac{1}{16}$	46.7	20 600	37.5	550	770
37..	C	Atlas	1 : 2 : 5	4.00	5′	$\frac{1}{32}$	51.4	33 700	37.5	900	590
38..	C	"	1 : 2 : 5	4.50	5′	$\frac{1}{32}$	51.4	38 500	37.5	1028	574
39..	G	"	1 : 3 : 6	4.75	5′	$\frac{1}{16}$	8.6	51 500	37.5	1375	890
40..	G	"	1 : 3 : 6	4.19	5′	$\frac{1}{16}$	8.6	43 800	37.5	1170	940
43..	S	"	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	3.87	5′	$\frac{1}{16}$	12.8	22 200	37.5	590	463
44..	S	"	1 : $2\frac{4}{10}$: $4\frac{6}{10}$	4.00	5′	$\frac{1}{16}$	12.8	23 000	37.5	612	625
61..	C	"	1 : 2 : 5	4.24	6′	$\frac{1}{32}$	51.4	23 400	54.0	433	330
62..	C	"	1 : 2 : 5	3.93	6′	$\frac{1}{32}$	51.4	21 200	54.0	392	334
63..	C	"	1 : 2 : 5	4.38	8′	$\frac{1}{32}$	51.4	26 400	96.0	275	162
64..	C	"	1 : 2 : 5	4.51	8′	$\frac{1}{32}$	51.4	27 250	96.0	283	156
65..	C	"	1 : 2 : 5	4.51	10′	$\frac{1}{32}$	51.4	51 500	150.0	343	196
66..	C	"	1 : $2\frac{4}{10}$: 6	4.56	9′	$\frac{1}{16}$	57.0	26 000	121.5	214	213
67..	C	Silica	1 : $1\frac{6}{10}$: 4	4.37	6′	$\frac{1}{32}$	51.4	23 600	54.0	437	417
68..	C	"	1 : $1\frac{6}{10}$: 4	4.44	6′	$\frac{1}{32}$	51.4	24 100	54.0	446	375
69..	C	"	1 : $1\frac{6}{10}$: $3\frac{7}{10}$	4.12	8′	$\frac{1}{32}$	51.4	22 000	96.0	228	195
70..	C	"	1 : $1\frac{6}{10}$: 4	3.75	8′	$\frac{1}{32}$	51.4	22 000	96.0	228	188
71..	C	"	1 : $1\frac{6}{10}$: 4	3.25	10′	$\frac{1}{32}$	51.4	18 700	150.0	125	114
72..	C	"	1 : $1\frac{6}{10}$: 4	4.19	10′	$\frac{1}{32}$	51.4	25 200	150.0	168	106
77..	S	"	1 : $2\frac{4}{10}$: 4	3.93	10′	$\frac{1}{16}$	12.8	39 800	150.0	265	252
78..	S	Atlas	1 : 3 : 5	4.50	10′	$\frac{1}{11}$	12.8	75 000	150.0	500	225
....	....	.....	.....	.....	....	....	.....	.....	Means..	528	421

A. L. Johnson

Table No. 4 gives, in the last two columns, a comparison between the theoretical strength of the various test panels with the actual results obtained from the data given by the author. The list includes all the tests that had been published at the time the table was prepared. In the theoretical determinations the assumption was made that the expanded metal was laid directly on the centers before the concrete was placed, thus putting the expanded metal as low as possible in the slabs. It will be seen, by comparing these columns, that, while many of the results agree very closely, in general, the actual results obtained were much below that which the theory demands; a condition which has already been referred to, and explained as probably being due to the improper location of the metal, and to the poor quality of the concrete used.

The values used for the modulus of elasticity are as follows:

$$\text{Expanded metal} = \frac{E_s}{\sec \frac{\Theta}{2}} = \frac{28\,500\,000}{1.11} = 25\,700\,000 \text{ lbs. per square inch.}$$

Concrete (stone)	1 : 3 : 6	2 000 000	"	"
" (gravel)	1 : 3 : 6	3 000 000	"	"
" (cinder)	1 : 2 : 3	550 000	"	"
" "	1 : 2 : 5	500 000	"	"
" "	1 : 3 : 6	450 000	"	"
" "	1 : 2 : 8	400 000	"	"

The values for the cinder concrete and expanded metal were obtained from tests made by the writer.*

Table No. 5 is a comparison of the theoretical with the observed values of the load-carrying capacity of some slabs made up by the writer about two years ago.

It will be seen that the theoretical values are all about the same, which is as they should be, as any of the mixtures of a thickness of 4 ins. were able to develop the tensile strength of the metal. The latter had not been well cut, and the value of $\frac{a^2}{d}$ for this metal was averaged at 0.02. In cutting the sheets out of a larger sheet, it was possible to get one more strand of metal in some sheets than in others, which would make a difference of about 10% in the strength of the slab. This will probably account for the discrepancies existing, though as they stand they give a very satisfactory check.

The author's formula (1) is very far from being the true expression of the total moment of resistance. The error made is in assuming that the safe working stresses C, T and S are attained at the same time, and this is far from being the case.

* The other values were obtained from Johnson's "Materials of Construction."

TABLE No. 5.—Showing Agreement Between Computed and Observed Values for the Strength of Expanded Metal Concrete Slabs. A. L. Johnson.

Proportions of mixture.	Thickness of slab, in inches.	Span, in feet.	Modulus of elasticity of the concrete.	Computed. Carrying capacity, in pounds per square foot uniformly distributed.	Observed. Value for same by actual test.	Remarks.
1 cement ..... 3 sand ..... 5 cinders.....	4	2⅔	500 000	1 460	1 110	May have had one strand less metal.
1 cement ..... 1 sand ..... 6 cinders.....	4	2⅔	400 000	1 430	1 340	
1 cement ..... 2 sand..... 5 cinders.....	4	2⅔	500 000	1 460	1 730	May have had one extra strand of metal.
1 cement ..... 1 sand..... 1 cinders.....	4	2⅔	550 000	1 470	1 400	
1 cement ..... 5 cinders.....	4	2⅔	450 000	1 460	1 650	
1 cement ..... 6 cinders.....	4	2⅔	400 000	1 430	1 350	
Mean	....	....	.....	1 452	1 430	

Slabs were supported on knife edges 32 ins. apart and broken by a load in the middle.

If the neutral axis were in the middle of the slab, T and C would be equal up to the time the concrete failed in tension, instead of T being one-fifth as much as C, as the author takes it.

After the concrete fails in tension on the underside there are only two terms left in the equation for the moment of resistance, *i. e.*, the first and last.

Now, this failure in tension occurs at about one-fourth of the ultimate strength of the construction, on the average, so that for three-fourths of the period of the test the author's formulas have no possible application. For the other fourth, C and T are nearly equal instead of T being equal to $\frac{C}{5}$.

To sum up: to correctly evaluate the moment of resistance it is necessary to know the location of the neutral axis, which can only be obtained by considering the relative moduli of elasticity of the metal and concrete used.

The author makes no mention of these functions, and, therefore, his equations, can have no theoretical or practical value.

Mr. Hill. GEORGE HILL, Assoc. M. Am. Soc. C. E.—Before answering the discussion, the author desires to present the results of a second series of tests made by him, in Philadelphia, in June, 1898. The cement used in this series was an American Portland extensively used for concrete slabs.

The load was applied in all cases on the full width of the center of the slab, through a wooden block, in such a manner as to avoid injuring the top surface of the concrete. The slabs were supported on steel shapes which did not injure the concrete in any way, and were 60 ins. apart. They were so hung as to be free to move, but did not move an appreciable amount. They were 11.50 ins. wide and of the thickness noted. No. 10 expanded metal, cut so as to leave no waste strands, was used, and was weighed in order to determine the exact sectional area of the metal at the point of greatest stress. The compressive strength was determined from cylinders 6 ins. in diameter and 6 ins. long. The proportions of the concrete were uniformly 1 cement, 2 sand and 5 cinders. The cinders were anthracite furnace cinders, granular in form, irregular in size and with a very small percentage of ash, apparently a better material than that used for the tests of Table No. 3. The mixing was done in a manner similar to that used in making commercial slabs in the locality, and was perhaps rather more thorough than the mixing of the first series of test slabs. The tamping was done with an iron rammer weighing a little over 9 lbs. The metal was uniformly placed in the bottom of the slab so that the bottom edge was just covered. The testing apparatus and gauges were similar to those used in the other tests. The load was applied slowly, and carried to the destruction of the material. The deflections were very carefully measured by means of a 2-ft. rule. The character of the concrete seemed to be better than that used in the first series of tests (Table No. 3), and showed a higher average of compressive strength. The author believes, however, that the shearing strength was actually less, since in every case in which metal was used the concrete was not strong enough to prevent the drawing in of the metal, oftentimes to so great an extent as to develop but a small percentage of the strength of the metal. The concrete between the strands of metal in every case was sheared off and dropped at the time the maximum load was reached; the shelling extending over an area of about one-half of the total area of the slab, being a maximum at the center and a minimum toward the ends, showing conclusively that the intensity of the tensile strain in the bottom slab varied from a maximum at the center to zero at the end. In every case failure first occurred as a crack on the under side of the concrete. When the ultimate load was reached this crack extended to the top of the concrete, and the opening varied from $\frac{1}{4}$ in. to $\frac{3}{4}$ in. In no case did the slab containing metal collapse. The strength developed, up to the

TABLE No. 6.

Test number.	Age, in days.	Thickness, in inches.	Square inches of metal.	Plaster Cracks.		1st Crack.		Percentage.	Maximum.		Ultimate.		Compressive strength, per square inch.	R.	Remarks.
				Load.	Deflection.	Load.	Deflection.		Load.	Deflection.	Load.	Deflection.			
1	34	3	0	.....	.....	.....	....	.....	.....	....	.....	.....	727	.....	All compression tests made six days later.
2	34	3	0	.....	.....	283	....	.....	.....	....	283	.....	576	246	
3	34	3	0.15	635	0.167	710	0.19	81.5	870	....	.....	.....	.....	758	
4	34	3	0.15	536	0.167	800	0.25	89.5	895	0.37	700	0.56	.....	780	Flaw in side.
5	34	3	0.31	520	0.167	1 550	0.50	100	1 550	0.50	.....	.....	628	1 350	
6	34	3	0.31	480	0.167	1 430	0.50	100	1 430	0.50	.....	.....	.....	1 220	
7	32	4	0	.....	.....	471	....	.....	471	....	.....	.....	.....	231	
8	34	4	0	.....	.....	376	....	.....	376	....	.....	.....	510	184	
9	32	4	0.15	1 050	0.167	1 175	0.19	100	1 175	0.19	895	0.75	.....	576	
10	34	4	0.15	925	0.167	1 035	0.19	97.8	1 060	0.25	800	0.68	665	520	
11	32	4	0.31	1 365	0.167	2 440	0.30	100	2 440	0.30	940	0.81	.....	1 196	
12	34	4	0.31	1 115	0.167	1 650	0.25	100	1 650	0.25	1 500	0.32	748	785	Flaw in side at bottom.
13	34	5	0	.....	.....	518	0.06	.....	518	0.06	.....	.....	.....	162	
14	34	5	0	.....	.....	590	0.06	.....	590	0.06	.....	.....	.....	185	
15	34	5	0.15	.....	.....	1 230	0.12	87.3	1 410	0.19	940	0.44	503	442	
16	34	5	0.15	1 465	0.167	1 640	0.19	100	1 640	0.19	1 410	0.30	396	514	
17	34	5	0.31	2 180	0.167	2 440	0.19	98.2	2 490	0.25	.....	.....	461	771	
18	34	5	0.31	1 890	0.167	2 120	0.19	93.8	2 260	0.25	1 975	0.56	530	708	
19	34	6	0	.....	.....	1 080	0.06	.....	1 080	0.06	.....	.....	.....	236	
20	34	6	0	.....	.....	1 130	0.06	.....	1 130	0.06	.....	.....	.....	243	
21	34	6	0.15	.....	.....	1 880	0.12	90	2 090	0.19	.....	.....	.....	546	
22	34	6	0.15	1 760	0.167	1 975	0.19	100	1 975	0.19	1 650	0.31	.....	516	
23	34	6	0.31	2 900	0.167	3 250	0.19	95.8	3 390	....	2 440	0.44	.....	885	
24	34	6	0.31	3 340	0.167	3 740	0.19	100	3 740	0.19	.....	.....	.....	978	
25	34	7	0	.....	.....	1 390	0.06	.....	1 390	0.06	.....	.....	.....	223	
26	34	7	0	.....	.....	1 450	0.06	.....	1 450	0.06	.....	.....	.....	232	
27	34	7	0.15	.....	.....	2 350	0.12	87.8	2 680	0.19	1 320	0.50	.....	428	Metal broke.
28	34	7	0.15	.....	.....	2 450	0.06	95.0	2 580	0.12	1 550	0.75	.....	415	" "
29	34	7	0.31	2 500	0.167	3 720	0.25	100	3 720	0.25	2 680	0.50	.....	595	
30	34	7	0.31	.....	.....	3 530	0.12	92.3	3 830	0.19	1 455	0.62	.....	613	
31	34	8	0	.....	.....	1 505	0.06	.....	1 505	0.06	.....	.....	.....	184	
32	34	8	0	.....	.....	1 740	0 06	.....	1 740	0.06	.....	.....	.....	214	
33	34	8	0.15	1 760	0.167	1 975	0.19	86.8	2 260	0.05	.....	.....	.....	277	
34	34	8	0·15	.....	.....	3 150	0·12	100	3 150	0.12	1 600	0.67	Average.	386	Metal broke after pulling together.
35	34	8	0.31	.....	.....	4 710	0.12	84.8	5 550	0.19	3 850	0.67	574.4	680	
36	34	8	0.31	5 360	0.167	5 880	0.19	100	5 880	0.19	2 770	1.00	.....	720	Metal broke after drawing together.
37*	..	3	0.65	2 680	0.17	4 700	0.50	.....	4 700	0.75	3 530	.....	.....	.....	Metal broke.
38*	..	3	0.65	3 150	0.17	4 930	0.44	.....	5 550	0.56	4 470	.....	.....	.....	Metal broke.

* In test No. 37 the metal drew together before breaking. In test No. 38 there was no drawing together until after the maximum was passed and a load of 5 180 lbs. was reached with a deflection of 0.81 in. The first crack appeared when an average of 95% of the strength was developed, the minimum being 81.5%, the maximum 100%, in 11 out of 21 cases. Tests Nos. 37 and 38 show the same general results.

* In tests Nos. 37 and 38 the slabs were made of the same mixture as the others, but were 3 ins. thick, 47½ ins. wide and with a span of 60 ins. No. 10 metal was used in the bottom and the loads were applied centrally.

Mr. Hill. time when plaster would crack, compared favorably with the strength developed in the first series. The values given under the heading R of Table No. 6 were computed by using the usual value of the moment of inertia of a rectangular section, $\frac{b\,d^3}{12}$. No attempt was made to compare the results with the formula suggested by the author, because the metal did not develop its proper strength. The results given under the heading "percentage" compare the load producing the first crack with the maximum load carried, in elucidation of the statement, made in the discussion by some of the members, that the concrete ordinarily cracks at one-quarter of the maximum load. This series of tests is of interest as showing other characteristics of the combination, and emphasizes the considerable reinforcing action of the metal. It also shows the desirability of using a concrete of high shearing strength.

Taking up now the discussion of the members, and answering Mr. Lesley, the author agrees with Mr. Tomkins and with Mr. Merritt, in the belief that no danger is to be apprehended from the use of limestone or from sulphur, for the reasons stated. The qualification that the concrete should be the best obtainable might be construed to mean stone concrete or cinder concrete made carefully and meeting the commercial conditions. The number of tests which have been made indicate that any concrete which can carry its own weight when the centers are struck, from seven to ten days after laying, is absolutely safe. Mr. Washburn's criticism of the mixing is probably well taken. The reason for permitting the mixing to be done in this way was that the author wished to test the slabs under commercial conditions. It is a matter for regret that Mr. Dunham did not mention at least one specific instance in support of his position. The *Transactions* of the Society show a great many instances where concrete and steel have been used in conjunction successfully. The only instances where concrete has not entirely prevented steel from oxidation, of which the author has knowledge, are those where the steel entered the concrete in such a manner as to lead moisture into it, the concrete being porous, and, therefore, not in intimate contact with the steel. The author has seen concrete slabs with expanded metal in them, which had been exposed to the weather for five years, in which absolutely no change had taken place in the expanded metal. Mr. Merriman's inquiries are answered in the second series of tests hereinabove mentioned. Mr. Waite's discussion of the commercial side entirely neglects the cost of the filling-in material between the beams, which is an item of serious amount, and further, it neglects the peculiarly advantageous features which may be derived from the fact that the entire slab may be made continuous, thereby avoiding the necessity of either beams

or girders. For a column spacing of 15 ft. centers each way, to carry a live load of 70 lbs. per square foot, there would be required: Mr. Hill.

9-in. I-beams, at 21 lbs., at 7 ft. 6 in. centers	630 lbs.
1 girder, 10-in., I, at 25 lbs	375 "
Total	1 005 lbs.

1 005 lbs. at 2 cents	$20.10	
Add for connections	2.01	
Floor arch blocks, 225 sq. ft., at 18 cents	40 50	
		$62.61
For expanded metal construction, 392½ lbs. of tie beams, at 2 cents	$5.85	
Connections	1.17	
Flooring, 225 sq. ft., at 16 cents	36.00	
		43.03
Saving, 31 per cent		$19.59

The point raised concerning deflection will be found fully answered in the paper.

Mr. J. B. Johnson's statements are not apparently verified by the tests, since, in no case, is the ultimate strength of the beams tested three times that developed when the concrete first cracks, which would seem to refute, in part, at least, the discussion published in *Engineering News*, October 21st, 1897. The author recognizes the difficulties confronting the development of a theoretical formula, but believes that a working formula has been deduced. The assumption that the steel reaches its working stress, or very near it, simultaneously with that of the concrete in tension seems to be borne out by the tests. Since the modulus of elasticity is unknown, variable and low, it is probable that, with a sufficient number of tests to indicate the proper value of the constants, the practical formula will give better results than a theoretical one. The author did not offer the formula as theoretically correct, but, as being practically so, and simply explained the way in which it was derived. Concerning the discussion by Mr. A. L. Johnson, the author wishes to state:

First.—No arch action of any amount could be set up in a slab of such relatively small thickness, in which concrete was under compression, through the action of a bending moment causing tension on the under side of the slab. The author has made a number of tests of concrete arches which he is not at liberty to publish, but which, with much greater rise, developed a very much less strength than the same amount of concrete for the same span made as a slab with expanded metal.

Mr. Hill.

Second.—The position of the expanded metal in the slab was carefully ascertained and is correctly stated in Table No. 3.

Third.—The author was conducting experiments to determine as far as possible the influence of expanded metal in the bottom portion of a concrete slab. He was endeavoring to make the tests in such a manner that they would be of general engineering value. This would not have been the case had the tests been conducted solely on the floor slabs in completed buildings.

Fourth.—The influence of the weather in retarding the set of the cement is, by the showing made by Mr. Johnson in Fig. 15, but trifling. It may be that the cylinders were injuriously affected, as they were so placed that they might have remained frozen for some little time.

Fifth.—Had Mr. Johnson used the paper for the basis of his criticism instead of referring in part to the same, and in part to the reports published in the technical journals, he would have made his discussion more easy to follow, and might have arrived at a different conclusion in regard to certain matters. In part, his comparison of test No. 5 with test No. 7 would probably have been omitted, since the author stated that the stone used in test No. 5 was irregular in size and too large, while in test No. 7 this defect was overcome by the admixture of gravel. In Table No. 4 the thickness given is, in general, the sum of columns *c* and *d* of Table No. 3. For purposes of calculation only, column *c* of Table No. 3 should have been used, as only thus could a proper comparison be made. The comparison should further have been made with the actual carrying capacity of the centrally applied load, and not with a converted distributed load; the numbering of the tests should have followed Table No. 3; the tests numbered 33, 34, 35, 36, in Table No. 4, would have been omitted; and the cement designated silica would have been called slag.

Sixth.—The assertion made in a number of places that as the percentage of aggregate increases, the strength of the concrete decreases in a nearly uniform ratio, is one which the author believes to be in error, both practically and theoretically. The maximum strength for any given aggregate will be obtained by the use of certain proportions of cement, sand and aggregate, depending on the character of the aggregate, which, though called by the same name, may vary very greatly in its composition. The thoroughness of the mixing, and the amount of water are also elements of great importance; as a consequence it is probably impossible to obtain any correct idea of the strength of the concrete from the simple statement that it is mixed in a certain proportion.

Seventh.—The error, contained in the statement that the ultimate strength of the combination of metal and concrete is nearly four times that developed at the time when the concrete first cracks, is disproved

by reference to the tests of Table No. 3, in which the minimum is 69.4%, the average, not 25%, but 91.5%, and the maximum is 100% in eleven cases out of thirty-nine (see also the percentages given in Table No. 6). The discussion, therefore, of the formula to be applied after the concrete cracks on the tension side is based on an erroneous assumption. Mr. Hill.

Eighth.—It seems to be generally admitted that the modulus of elasticity of commercial concrete is not yet known. The lack of uniformity in results, which must be known to every one using and testing concrete, shows how it must vary. Further than this, it is not known whether the modulus of elasticity for concrete is the same for both compression and tension; if it is, then the concrete should be as liable to fail first on one side as the other. If it is not, then all of the formulas given by Mr. Johnson are wrong, for they are based on the assumption that the E_c has only one value. If E_c has two values, then the formulas are wrong, if only because they do not recognize these two values. E may be expressed by the fraction $\frac{\text{stress}}{\text{deformation}}$. Now, it is known that the stresses are the same on both sides of the neutral plane within the elastic limit of the material, that if E is the same on both sides of the neutral plane, that this plane is in the center of gravity axis of the figure; and since the stress is the same and E is the same, the deformation must be the same, and, consequently, that strain which causes a deformation visible as rupture on the tension side must cause a movement of the particles on the compression side amounting to rupture, though invisible.

Ninth.—The literature on the subject would have been enriched, perhaps, had Mr. Johnson given the details of the experiments on which he bases the value of the modulus used by him, since there would then have been an opportunity to discuss them. The table on page 656 expresses the value of the modulus in "pounds per square inch."

The modulus is expressed by the formula $E = \frac{\text{stress}}{\text{deformation}}$, which is an absolute quantity and is not "pounds per square inch."

For the expanded metal the modulus remains the same, regardless of the position of the strands, and it is incorrect to vary it according to the position. A proper method would be to introduce a member in the formulas which would represent the condition that exists, namely, that the stands of metal make an angle with the lines of strain in the slab. Even, then, the formulas would not be entirely correct, since the strands join at what are commercially called bridges, at which points the axis of the bridge is parallel with the lines of strain, and the resultants of the pull in the strands at right angles to the lines of strain neutralize each other.

Mr. Hill. As the bridge is the weak point, a reduction of the strength of the expanded metal should be made therefor, but this should not be made by changing E.

Tenth.—Mr. A. L. Johnson is credited by Professor J. B. Johnson with the formulas published in *Engineering News*, October 21st, 1897. If this formula be expressed in the notation shown on Fig. 20, it will read $y_1 = \sqrt{\frac{2\,a\,y_2}{b}\,\frac{E_s}{E_c}}$, in which a is the area of the steel bars; this figure differs from the one used in the article referred to, and, by examining Fig. 20, it will be seen that the equation should read $y_1 = \sqrt{\frac{3\,a\,y_2}{2\,b}\,\frac{E_s}{E_c}}$, to be numerically correct, since it appears to be derived from the equation $\frac{2}{3}\,b\,y_1^2\,E_c = a\,y_2\,E_s$.

Eleventh.—Table No. 5, given by Mr. A. L. Johnson, as a practical proof of the correctness of his formulas, is, by his own statements, discredited; for the experiments were conducted so carelessly that he does not know the exact amount of metal used in the slabs.

Twelfth.—The value of the constants C and I must be based on experiments made by the manufacturer or engineer. Commercial considerations will limit the use of metal and the thickness of the slabs so as to be always safe. The results of 94 tests show that nearly every one of Mr. A. L. Johnson's statements are erroneous, and help to discredit his formulas.

To sum up: the author believes that he has shown the commercial value of the combination, has indicated the precautions that should be taken in using it, has added materially to the knowledge of the action of concrete and steel in combination, and has offered a formula, not new in form, but of practical value, for use until something better can be obtained, and the only formula based on a knowledge of the actual performance of the material.